Variable-range hopping conductivity in a strong magnetic field

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During nonresonant subbarrier tunneling in a transverse magnetic field, scattering by impurities causes the r dependence of the transparency to become $\exp(-2r/b)$, in contrast with the dependence $\exp(-r^2/2\lambda^2)$ without scattering. This change explains the experimentally observed temperature dependence of the variable-range hopping conductivity in a magnetic field.

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Tokumoto, Mansfield, and Lea[1] recently studied the hopping conductivity with a variable hopping range (variable-range hopping) of n-type InSb samples at temperatures T down to 0.03 K and in magnetic fields H up to 40 kOe. They found that the temperature dependence of the resistivity can be described well by

$$\rho = \rho_0 \exp(T_0/T)^{1/2} . \tag{1}$$

Mott[2] derived the law

$$\rho = \rho_0 \exp(T_0/T)^x \tag{2}$$

with $x = 1/4$ for the case of a constant state density $g(\epsilon)$ near the Fermi level and with donor wave functions falling off in proportion to $\exp(-ra)$. It was shown later[3] that there is actually a Coulomb gap near the Fermi level; i.e., we have $g(\epsilon) \simeq \kappa^3 e^{-6} \epsilon^2$, where the energy ϵ is reckoned from the Fermi level, e is the electron change, and κ is the dielectric function. According to Ref. 3, the Coulomb gap leads to expression (1) with $T_0 = \beta e^2/(k\kappa a)$, where k is the Boltzmann constant, and $\beta \simeq 2.7$ (Ref. 4). A strong magnetic field affects variable-range hopping because the wave functions are compressed. Far from the donor, which is at the origin of a cylindrical coordinate system with z axis running parallel to H, the wave function is

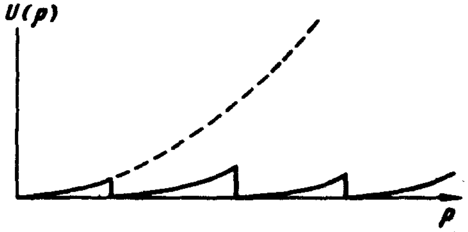

FIG. 1.

$$\psi(\rho, z) \propto \exp\left\{-\frac{\rho^2}{4\lambda^2} - \frac{|z|}{a(H)}\right\}, \tag{3}$$

where $\lambda = c\hbar/(eH)$ is the magnetic length, $a(H) = \hbar[2mE(H)]^{-1/2}$, m is the effective mass, and $E(H)$ is the binding energy of an electron at a donor in the field H. The reason for the rapid decay of wave function (3) in the transverse direction is that the magnetic field creates a potential barrier $U(\rho)$ near the donor, and this barrier increases in proportion to ρ^2 with distance from the z axis (the dashed curve in Fig. 1). The use of the wave functions in (3) changes the temperature dependence of the variable-range hopping. With the Coulomb gap ignored, the value $x = 1/3$ was derived in Ref. 5. With a Coulomb gap, the value should be $x = 3/5$, according to the calculations of Tokumoto *et al.* However, this conclusion is contradicted by the experiments of Tokumoto *et al.*, which very accurately yield the value $x = 1/2$.

In the present letter we wish to show that although the wave function of an isolated donor is of the form in (3) and does completely determine the hopping conductivity in the case of hops between nearest neighbors,[6] it cannot be used for describing variable-range hopping. In the latter case, we need the wave function for distances ρ greater than the average distance between donors, $R \equiv N^{-1/3}$, where N is the donor concentration. In the case of tunneling away from its donor, an electron may then be scattered repeatedly by other donors, and in each of these events the center of the Landau oscillator of this electron is "displaced"; i.e., the potential barrier created by the magnetic field is "pushed away" (Fig. 1). As a result, the barrier height ceases to increase monotonically with increasing ρ, and the wave function becomes

$$\psi(\rho, z) \propto \exp\left\{-\frac{|z|}{a(H)} - \frac{\rho}{b(H, N)}\right\}, \tag{4}$$

where $b(H, N)$ is a length that will be found below. The use of wave function (4) to describe variable-hopping leads to expression (1) with $T_0 = \beta_1 e^2 (k\kappa)^{-1}[a(H)b^2]^{-1/3}$, where β_1 is a numerical factor. The contradiction with the experimental results of Tokumoto *et al.* regarding the value of x is thus resolved.

To derive (4) we work from the Hamiltonian for the impurity band of a lightly doped semiconductor in the representation of wave functions ψ_i^0 of the type in (3) which correspond to isolated donors:

$$\mathcal{H} = \sum_i \epsilon_i a_i^+ a_i + \sum_{i \neq j} V_{ij} a_i^+ a_j, \tag{5}$$

where ϵ_i is the energy of the state at the donor i, a_i^+ is the operator which creates an electron in this state,

$$V_{ij} = V_0 \exp\left\{ -\frac{x_{ij}^2 + y_{ij}^2}{4\lambda^2} - \frac{|z_{ij}|}{a(H)} \right\} \exp\{-i\Phi_{ij}\} \tag{6}$$

is the matrix element for the transition $i \to j$, $x_{ij} = x_j - x_i$, $\mathbf{r}_i = (x_i, y_i, z_i)$ is the coordinate of donor i, and $\Phi_{ij} = (e/\hbar c)\mathbf{H}[\mathbf{r}_i \mathbf{r}_j]$. We assume that the overlap of donors is very slight, so that $|V_{ij}| \ll |\epsilon_j - \epsilon_i|$, and we consider the long-range behavior of the eigenfunction $\psi_1(\mathbf{r})$ of Hamiltonian (5). This eigenfunction has an energy of approximately ϵ_1 and is basically localized near donor 1. For this purpose we write $\psi_1(\mathbf{r})$ as the series

$$\psi_1(\mathbf{r}) \simeq \psi_1^0(\mathbf{r}) + \sum_i \frac{V_{1i}\,\psi_i^0(\mathbf{r})}{\epsilon_1 - \epsilon_i} + \sum_{i \neq j} \frac{V_{1i} V_{ij}\,\psi_j^0(\mathbf{r})}{(\epsilon_1 - \epsilon_i)(\epsilon_1 - \epsilon_j)} + \dots, \tag{7}$$

whose terms describe the tunneling of an electron from the point $\mathbf{r}_1$ (below we set $\mathbf{r}_1 \equiv 0$) to the point $\mathbf{r}$ without scattering, with one scattering event, with two scattering events, etc. The series in (7) differs from a perturbation-theory series in that we have discarded terms which describe the return of an electron to the same point and which, in particular, arrange the correct normalization of $\psi_1(\mathbf{r})$. In the terminology of Ref. 7, series (7) describes nonresonant tunneling. Under the conditions $R \gg \lambda$ and $\mathbf{r} \gg R$ in the direction perpendicular to H, the various terms in (7) differ exponentially, and the argument of the exponential function in $\psi_1(\mathbf{r})$ is determined by the optimum tunneling path for the given arrangement of donors. The modulus of this argument is much smaller than that of $\psi_1^0(\mathbf{r})$. To show this, we first consider the simpler two-dimensional problem of the donor wave functions in a very thin film in a magnetic field which is perpendicular to the film. In this case we have $V_{ij} \propto \exp(-r_{ij}^2/4\lambda^2)$, and the most probable optimum tunneling path to the point $\mathbf{r}$ in the plane of the film will consist of $n \simeq \mathbf{r}/R$ "steps," each with a length of order R (Fig. 2). For each step we have $V_{ij} \propto \exp(-R^2/4\lambda^2)$ and thus

$$\psi_1 \propto \exp(-nR^2/4\lambda^2) = \exp(-r/b), \tag{8}$$

where $b = s\lambda^2 R^{-1}$, and s is a numerical factor. In the case $\mathbf{r} \gg R$ we obviously have $\psi_1(r) \gg \psi_1^0(r)$.

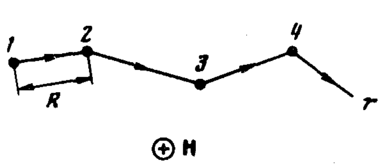

FIG. 2.

In the three-dimensional case at $\mathbf{r} = (\rho,0,0)$ the vector $\mathbf{r}_{ij}$ of a typical step usually lies within the surface

$$\frac{x_{ij}^2 + y_{ij}^2}{4\lambda^2} + \frac{|z_{ij}|}{a(H)} \leqslant \xi, \qquad x_{ij} > 0, \tag{9}$$

where ξ is determined by the condition $N\Omega(\xi) \simeq 1$, and $\Omega(\xi) = (\pi/2)\xi^2 a\lambda^2$ is the volume bounded by this surface. Some typical values are $V_{ij} \simeq V_0 \exp(-\xi)$ and $n \simeq \rho/\lambda\sqrt{\xi}$. As a result, $\psi_1(\mathbf{r})$ is determined by (4), where $b(H,N) = t\lambda^{3/2}a^{1/4}R^{-3/4}$, and t is a numerical factor. Under the conditions of Ref. 1, the field H is only a few times greater than the field H_c at which the metal-insulator transition occurs; i.e., $N\lambda^2 a \lesssim 1$. Here $b \simeq \lambda$. As $H \to H_c$, with $a(H) \propto (1 - H/H_c)^{-\nu}$, $b(H)$ should increase in proportion to $\lambda(1 - H/H_c)^{-\nu}$, and we should have $\psi_1 \propto \exp\{-(z^2/a^2 + \rho^2/b^2)^{1/2}\}$.

We wish to emphasize that the effect of subbarrier scattering on tunneling in a transverse magnetic field which has been described here is a completely general effect, in our opinion. In particular, the transverse nonresonant tunneling transparency of a film with impurities in a magnetic field directed parallel to the surface of the film should depend on the film thickness d, in accordance with $\exp(-2d/b)$, where b depends on H and N, rather than in accordance with $\exp(-d^2/2\lambda^2)$, as for a pure film.

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