Parametric decay of electromagnetic waves in a magnetized vacuum

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The possibility of parametric decay of an electromagnetic wave into oppositely moving waves in a vacuum with a strong magnetic field is demonstrated. It is found that this nonlinear effect in a magnetized vacuum is the most promising effect for experimental observations.

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In a magnetic field $\mathbf{B}_0 = (B_{0x}, B_{0y}, 0)$, comparable in magnitude to $B_c = m^2c^3/e\hbar$ (e and m are the charge and mass of the electron), the vacuum is a birefringent medium.[1] Here, two normal, linearly polarized, electromagnetic waves exist with wave vector $\mathbf{k}$: the $\|$ wave, for which the electric field $\mathbf{E}\perp\mathbf{k}$, and $\mathbf{E}\perp\mathbf{B}_0$ and the $\perp$ wave, for which the perturbation of the magnetic induction is $\mathbf{B}'\perp\mathbf{k}$, and $\mathbf{B}'\perp\mathbf{B}_0$. The nonlinearity of the vacuum in strong fields makes possible parametric decay: splitting of γ quanta in a magnetic field of the type $\gamma_\| \to \gamma_\perp + \gamma_\perp$.[2] Since the phase velocities of normal waves

$$V_\| = c\left(1 - \frac{2a}{45\pi} B_{0y}^2/B_c^2\right), \quad V_\perp = c\left(1 - \frac{7a}{90\pi} B_{0y}^2/B_c^2\right) \tag{1}$$

(where $a = e^2/\hbar c$) differ little from the velocity of light c, it can be shown that the laws of conservation of energy and momentum of photons or, using another language, the matching conditions for the interacting quasimonochromatic waves

$$\mathbf{k}_\| = \mathbf{k}_{\perp 1} + \mathbf{k}_{\perp 2}, \quad \omega_\|(\mathbf{k}_\|) = \omega_\perp(\mathbf{k}_{\perp 1}) + \omega_\perp(\mathbf{k}_{\perp 2}) \tag{2}$$

can be satisfied if the wave vectors $\mathbf{k}_{\perp 1,2}$ are oriented at small angles to the direction $\mathbf{k}_\|$.[2]

In this paper we want to point out the possibility of the decay of the $\|$ wave into oppositely moving $\perp$ waves. Such decay in some respects is more significant than the process examined in Ref. 2.

The field equations including the radiation corrections have the form

$$\begin{gathered} \operatorname{div}\mathbf{B} = 0, \quad \operatorname{div}(\mathbf{E} + 4\pi\mathbf{P}) = 0 \\ \operatorname{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \quad \operatorname{rot}(\mathbf{B} - 4\pi\mathbf{M}) = \frac{1}{c}\frac{\partial}{\partial t}(\mathbf{E} + 4\pi\mathbf{P}). \end{gathered} \tag{3}$$

Here the polarization $\mathbf{P} = \partial L'/\partial\mathbf{E}$ and magnetization $\mathbf{M} = \partial L'/\partial\mathbf{B}$ are expressed in terms of the correction to the Lagrangian density of the electromagnetic field[1]:

$$L = \frac{aB_c^2}{8\pi^2}\int_0^\infty \frac{\exp(-\eta)}{\eta^3}\left[-(\eta a \operatorname{ctg}\eta a)(\eta b \operatorname{cth}\eta b) + 1 - (a^2 - b^2)\eta^2/3\right]d\eta, \tag{4}$$

where $a = -i[(F+iG)^{1/2} - (F-iG)^{1/2}]/\sqrt{2}B_c$, $b = [(F+iG)^{1/2} + (F-iG)^{1/2}]/\sqrt{2}B_c$, $F = (\mathbf{B}^2 - \mathbf{E}^2)/2$, $G = (\mathbf{BE})$. It is not difficult to obtain from (3) the system of equations for waves of two polarizations traveling along the x axis:

$$\begin{aligned} \frac{\partial^2 B_y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 B_y}{\partial t^2} &= 4\pi\left(\frac{\partial^2 M_y}{\partial x^2} + \frac{\partial^2 P_z}{\partial x \partial t}\right), \\ \frac{\partial^2 B_z}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 B_z}{\partial t^2} &= 4\pi\left(\frac{\partial^2 M_z}{\partial x^2} - \frac{\partial^2 P_y}{\partial x \partial t}\right). \end{aligned} \tag{5}$$

The conditions (2) can be satisfied for quasimonochromatic waves propagating along the x axis if

$$\mathbf{k}_{\parallel} = (k_0, 0, 0),\ \mathbf{k}_{\perp 1} = (k_+, 0, 0),\ \mathbf{k}_{\perp 2} = (-k_-, 0, 0),$$
$$k_0 = k_+ - k_-,\quad \omega_{\parallel}(k_0) = \omega_{\perp}(k_+) + \omega_{\perp}(k_-). \tag{6}$$

It is easy to show that the bucking wave is in this case the low-frequency wave: $\omega_\perp(k_-) \ll \omega_\parallel(k_0)$, since

$$k_\perp = \frac{V_\parallel - V_\perp}{2c} k_0 = \frac{a}{60\pi} (B_{0y}/B_c)^2 k_0 \ll k_0 . \tag{7}$$

For $B_{0y} \sim 0.1 B_c$ we obtain $k_- \sim 10^{-6} k_0$. Thus, the properties of the spectra of the $\parallel$ waves will be carried over in the process of parametric decay far into the low-frequency region and will be manifested in the spectrum of $\perp$ polarized waves.

Expression (4) was obtained for slowly varying fields (characteristic frequency $\omega \ll mc^2/\hbar$) in the first approximation with respect to $a \ll 1$.

Calculating **P** and **M** in this approximation, we can ignore the effect of radiation corrections to the wave field by representing the total field in the form

$$\mathbf{B} = \mathbf{B}_0 + \mathbf{B}',\quad B'_x = E_x = 0,$$
$$B'_y = \mathrm{Re}\,[b_0 \exp(i\omega_0 t - i k_0 x)],\quad E_z = -B'_y,$$
$$B'_z = \mathrm{Re}\,[b_+ \exp(i\omega_+ t - i k_+ x) + b_- \exp(i\omega_- t + i k_- x)], \tag{8}$$
$$E_y = \mathrm{Re}\,[b_+ \exp(i\omega_+ t - i k_+ x) - b_- \exp(i\omega_- t + i k_- x)],$$

where $b_0(t)$ and $b_\pm(t)$ are complex amplitudes of the $\parallel$ and $\perp$ waves. We shall examine the interaction of waves in the approximation of weak nonlinearity ($|b_0| \ll |\mathbf{B}_0|$, $|b_\pm| \ll |\mathbf{B}_0|$). Assuming that $|\mathbf{B}_0| \ll B_c$, we shall make use of the expansion of L' for weak fields ($F, G \ll B_c$):

$$L' = \frac{a}{360\pi^2} (4F^2 + 7G^2)/B_c^2 . \tag{9}$$

Substituting (8) and (9) into Eq. (5) and averaging with respect to the rapid oscillations with frequencies ω_0, $\omega_\pm$, we obtain equations for the slowly varying complex amplitudes

$$\frac{db_0}{dt} = \frac{aB_{0y} k_0}{60\pi B_c^2} b_+ b_- ,\quad \frac{db_\pm}{dt} = \frac{i a B_{0y} k_\pm}{60\pi B_{0y}^2} b_0 b_\pm^* . \tag{10}$$

For a given amplitude b_0, these equations characterize the parametric instability of the $\perp$ perturbations with amplitudes $b_\pm$, whose increment is

$$\gamma_0 = \frac{aB_{0y}|b_0|}{60\pi B_c^2} c\sqrt{k_+ k_-} \simeq \left(\frac{a}{60\pi}\right)^{3/2} \frac{B_{0y}^2 |b_0|}{B_c^3} ck . \tag{11}$$

An analogous calculation for the decay of the $\parallel$ wave into $\perp$ waves traveling at small angles to it requires an expansion of L' up to cubic terms in F and G.[1] Using the equations for the coupled $\parallel$ and $\perp$ waves, obtained in such an approximation in Ref. 3, it is easy to obtain the corresponding increment:

$$\gamma_c = \frac{13\,a}{630\,\pi}\,ck_0\,\frac{B_{0y}^3\,|\,b_0\,|}{B_c^4} = \frac{52}{21}\left(\frac{15\,\pi}{a}\right)^{1/2}\frac{B_{0y}}{B_c}\,\gamma_0\,. \tag{12}$$

According to (12), the decay of the $\parallel$ wave into oppositely moving $\perp$ waves in weak fields $B_{0y} < 5\times 10^{-3}\,B_c$ occurs more rapidly than the decay, examined in Ref. 2, into $\perp$ waves with similar directions of propagation.

It is also important to note the characteristic features of the transformation of waves examined above: A strong frequency shift and a change in the polarization and direction of propagation are very important for observing this effect, since these characteristics make it possible, in principle, to separate the weak wave, produced as a result of parametric decay, against the background formed by the main radiation.

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