Spontaneously broken complete relativity

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It is suggested that the equations of the theory should have a spontaneously broken invariance with respect to the complex Poincaré group. The properties of the corresponding Goldstone field are discussed.

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In the equations of the theory as it exists today, not all of the velocities (including those higher than the velocity of light) of the motion as a whole are equivalent. This circumstance is usually explained on the basis that the relative velocity of any observers is lower than the velocity of light. The latter fact, however, is a property of not only the equations but also of the ground state (the vacuum). Since the symmetry of the equations can be higher than that of the ground state, it seems extremely relevant to expand the symmetry of the equations of complete relativity, i.e., to the equivalence of all velocities (except the velocity of light). Under ordinary conditions, the additional symmetry must be broken spontaneously if the relative velocities are to be below the velocity of light.

The Lorentz transformation matrices corresponding to velocities higher than the velocity of light contain imaginary elements. If we are to combine such transformations with real transformations in a single group, we must introduce as the symmetry group a 12-parameter complex Lorentz group [one of the classical complex groups, SO(4,C); see Ref. 1, for example], i.e., the set of all complex linear (unimodular transformations of the four coordinates x^i which leave the scalar product $x_i x^i$ invariant. The introduction of complex Lorentz transformations and thus of complex velocities makes it possible to "circumvent" the singularity $v = c$ in the complex plane. Complex translations must be incorporated along the real translations in order to achieve the group property. Complete relativity thus reduces to invariance with respect to the 20-parameter complex Poincaré group (the complex Lorentz group plus complex translations), broken to a 10-parameter real subgroup. In practice, of course, it is necessary to find a realization of the complex group in the real, physical space-time, eliminating the complex character of the coordinates by introducing appropriate Goldstone fields.

We introduce the spinor coordinate $\zeta = \mathbf{x}\vec{\sigma} + x^{\circ}$, and we write the general complex Poincaré transformation $L(\beta, \lambda, r)$ in the form $\zeta' = \beta r \zeta r^{+} + \lambda$, where β and r are unimodular matrices, and $\lambda = \lambda_1 + i\lambda_2, \lambda_1$, and λ_2 are Hermitian matrices. With $\lambda_2 = 0$ and $\beta = 1$ we find a real transformation. In accordance with the general theory of nonlinear realizations of a symmetry,[2-4] we introduce the two Goldstone matrix fields $B(x)$ and $\Omega(x)$, whose transformation law is determined by the condition $L(\beta, \lambda, r)L(\mathrm{B}, \Lambda, 1) = L('\mathrm{B}, '\Lambda, 1)L(1, 0, \tilde{r})$ with some $\tilde{r}$. In this case we find

$$'\mathrm{B}('x) = \beta r \mathrm{B}(x) r^{-1}, \quad '\Lambda('x) = \beta r \Lambda(x) r^{+} + \lambda, \tilde{r} = r . \tag{1}$$

The covariant differentials $D\mathrm{B} = \mathrm{B}^{-1} d\mathrm{B}$ and $D\Lambda = \mathrm{B}^{-1} d\Lambda$ of the Goldstone fields transform as Lorentz tensors: $'(D\mathrm{B}) = rD\mathrm{B}r^{-1}$ and $'(D\Lambda) = rD\Lambda r^{+}$.

We switch to a tensor notation, setting

$$\Lambda(x) = x^{\circ} + i a^{\circ}(x) + \vec{\sigma}(\mathbf{x} + i\mathbf{a}(x)), \quad \lambda = \lambda_1^0 + i\lambda_2^0 + \vec{\sigma}(\vec{\lambda}_1 + i\vec{\lambda}_2),$$

$$\mathrm{B}(x) = f_s + \sigma_a\left(\frac{1}{2} e_{\alpha\beta\gamma} f^{\beta\gamma} + i f^{0\alpha}\right), \quad \beta = \varphi_s + \sigma_\alpha\left(\frac{1}{2} e_{\alpha\beta\gamma} \varphi^{\beta\gamma} + i\varphi^{0\alpha}\right).$$

Here $a^i(x)$ and $\lambda^i{}_{1,2}$ are real 4-vectors, $f^{ik}(x)$ and φ^{ik} are antisymmetric real tensors, and the scalar f_s is expressed in terms of f^{ik} by the condition $\det \mathrm{B} = 1$:

$$f_s \equiv f_{s1} + i f_{s2} = \left(1 + \frac{1}{2} f_{ik} f^{ik} + i e_{iklm} f^{ik} f^{lm}\right)^{1/2}.$$

In precisely the same manner, $\varphi_s = \varphi_{s1} + i\varphi_{s2}$ can be expressed in terms of φ^{ik}.

By virtue of (1), the transformation laws for the Goldstone fields $a^i(x)$ and $f^{ik}(x)$ and the coordinates x^i are

$$'x^i = \varphi_{s1} x^i - \varphi_{s2} a^i(x) - \varphi^i{}_k a^k(x) - \frac{1}{2} e^{iklm} \varphi_{kl} x_m + \gamma^i{}_1,$$

$$'a^i('x) = \varphi_{s1} a^i(x) + \varphi_{s2} x^i + \varphi^i{}_k x^k - \frac{1}{2} e^{iklm} \varphi_{kl} a_m(x) + \gamma^i{}_2$$

$$'f^{ik}('x) = f_{s1} \varphi^{ik} + \varphi_{s1} f^{ik}(x) - \frac{1}{2} e^{iklm}\left(f_{s2}\varphi_{lm} + \varphi_{s2} f_{lm}\right) + e^{iklm} \varphi^n{}_m f_{ln}.$$

The covariant differentials are

$$Dx^i = f_{s1} dx^i - f_{s2} da^i + f^i{}_k da^k + \frac{1}{2} e^{iklm} f_{kl} dx_m,$$

$$Da^i = f_{s1} da^i + f_{s2} dx^i - f^i{}_k dx^k + \frac{1}{2} e^{iklm} f_{kl} da_m,$$

$$Df^{ik} = f_{s1} df^{ik} - \frac{1}{2} f_{s2} e^{iklm} df_{lm} - e^{iklm} f^n{}_m df_{ln}.$$

Regardless of its tensor structure, the Lagrangian of any Lorentz field becomes invariant with respect to a complex group if the ordinary derivatives ∂_i in it are replaced by covariant derivatives, $D_i = h^k{}_i \partial_k$, where $h^i{}_k$ form the matrix which is the inverse of

$$\bar{h}^i{}_k \equiv \frac{Dx^i}{dx^k} = f_{s1}\,\delta^i_k - f_{s2}\,\frac{\partial a^i}{\partial x^k} + f^i{}_e\,\frac{\partial a^l}{\partial x^k} + \frac{1}{2}\,e^{jmn}{}_k f_{mn}\,.$$

The derivatives D_i satisfy the commutation relations $[D_i D_k] = \Delta^l{}_{ik} D_l$, where

$$\Delta^l_{ik} = \bar{h}^l{}_m \left(h^n{}_i\,\frac{\partial h^m{}_k}{\partial x^n} - h^n{}_k\,\frac{\partial h^m{}_i}{\partial x^n} \right)$$

is obviously a Lorentz tensor.

Gauge fields require a special examination. The gauge transformation of the electromagnetic potential is $A_i \to A_i + D_i\chi$. The gauge-invariant fields are $F_{ik} = D_i A_k - D_k A_i - \Delta^l_{ik} A_l$. The Lagrangian of the electromagnetic field is, as usual, proportional to $F_{ik} F^{ik}$.

The quantities $(Da_i/Dx^k) - (Da_k/Dx^i)$ form a Lorentz tensor. It should vanish in the minimum realization of the complex symmetry. Here f_{ik} is expressed in terms of the a^i in a invariant way. In the linear aproximation we have

$$\frac{Da_i}{Dx^k} - \frac{Da_k}{Dx^i} \approx \frac{\partial a_i}{\partial x^k} - \frac{\partial a_k}{\partial x^i} - 2f_{ik}\,,\quad f_{ik} = \frac{1}{2}\left(\frac{\partial a_i}{\partial x^k} - \frac{\partial a_k}{\partial x^i} \right). \tag{2}$$

Let us examine the properties of a weak Goldstone field $a^i(x)$. Its interaction with Lorentz fields is determined by the quantities $h^i{}_k = \delta^i_k - \frac{1}{2} e^{imn}{}_k f_{mn}$, i.e., by the field f_{ik}. The vector field a^i has no real meaning; it serves as a potential. The scalar fields or, more generally, the fields in whose Lagrangian the operators ∂_i contract only with each other do not interact with the field f_{ik}. The same is true of the electromagnetic field. To see this, we transform to an equivalent description, carrying out the transformation

$$\tilde{\Phi}(x) = \left\{ 1 + \frac{1}{2}\left(h^i{}_k - \delta^i_k \right) L_i{}^k \right\} \Phi(x), \tag{3}$$

for all the Lorentz fields $\Phi(x)$, where L_{ik} are the generators of the real Lorentz group. For the new fields $\tilde{F}_{ik}$ and potenials $\tilde{A}_i$ of the electromagnetic field we have $\tilde{F}_{ik}\tilde{F}^{ik} = F_{ik}F^{ik}$, $\tilde{F}_{ik} = \partial_i \tilde{A}_k - \partial_k \tilde{A}_i$, on the basis of which we assume the following: The Lagrangian of the spinor field is, after transformation (3) (we will now omit the tilde),

$$L = \frac{i}{2}\left\{ \bar{\psi}\gamma^i(\partial_i + ieA_i)\psi - (\partial_i - ieA_i)\bar{\psi}\gamma^i\psi + i\,\frac{\partial f^{ik}}{\partial x^k}\,\bar{\psi}\gamma^5\gamma_i\psi \right\} - m\bar{\psi}\psi. \tag{4}$$

In the linear theory, by virtue of (2) and the relation $Df_{ik} \simeq df_{ik}$, the most general form of the Lagrangian reduces to the single term

$$L = -\frac{1}{16\pi G}\,g^{lm}\,\frac{\partial f^{ik}}{\partial x^l}\,\frac{\partial f_{ik}}{\partial x^m}\,, \tag{5}$$

where G is some interaction constant. The field equations which follow from (2), (4), and (5) under conditions such that the momenta of the spinor particles vanish ($\mathbf{p} \to 0$) are

$$\Box(\dot{\mathbf{e}} - \operatorname{rot}\mathbf{b}) = -4\pi G(\ddot{\mathbf{s}} + \operatorname{rot}\operatorname{rot}\mathbf{s}), \quad \operatorname{rot}\mathbf{e} + \dot{\mathbf{b}} = 0\ ; \tag{6}$$
$$\Box\operatorname{div}\mathbf{e} = -4\pi G\operatorname{div}\dot{\mathbf{s}}, \quad \operatorname{div}\mathbf{b} = 0\ .$$

Here $e_\alpha = f_{0\alpha}$, $e_{\alpha\beta\gamma} b_\gamma = -f_{\alpha\beta}$, and $\mathbf{s} = \frac{1}{2}\psi^* \vec{\sigma} \psi$ is the spin density. The Hamiltonian of a spinor particle under the same conditions is

$$H = {}^1/_2\, \vec{\sigma}(\operatorname{rot}\mathbf{b} - \dot{\mathbf{e}})\ . \tag{7}$$

In the static case we have $\Delta\mathbf{b} = -4\pi G\operatorname{rot}\mathbf{s}$; i.e., the spins interact through the field $\operatorname{rot}\mathbf{b}$ as magnetic dipoles interact through the magnetic field, but with the unknown constant G.

The assumption of complete relativity thus leads in an unambiguous way to the existence of a massless Goldstone field which, in the linear approximation, has no manifestations of any sort in classical mechanics and whose sole macroscopic manifestation is the presence of a nonelectromagnetic long-range interaction of bodies having a nonvanishing average spin density. In principle, an extremely simple experiment to detect such a field might be to observe the precession of the magnetization of one ferromagnet in the Goldstone field of another, which is completely shielded from the first in the electromagnetic sense by currents (for example, superconducting currents).

Finally, we note that raising the temperature should lead to a complete restoration of this symmetry, as for other spontaneously broken symmetries. Such a restoration, possible in the early stages of a hot universe, for example, should be accompanied by the appearance of a complex nature, i.e., by a doubling of the dimensionality of the space-time, as is clear from the discussion above.

[1]M. A. Naĭmark, Teoriya predstavleniĭ grupp (Theory of Group Representations), Nauka, Moscow, 1976, p. 275.
[2]C. Isham, A. Salam, and J. Strathdee, Ann. Phys. (NY) **62**, 3 (1971).
[3]D. V. Volkov, Fiz. Elem. Chastits At. Yadra **4**, 3 (1973) [Sov. J. Part. Nucl. **4**, 1 (1973)].
[4]A. B. Borisov and V. I. Ogievetskiĭ, Teor. Mat. Fiz. **21**, 329 (1974).

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