Mass relations for fermions and the mass of the *t* quark

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The mass relations for fermions in grand unification models are examined. A restriction on the mass of the t quark, $m(t) \leqslant 24.5$ GeV, is found under the assumption of a linear relationship between the mass matrices of the charged leptons and quarks with charges of 2/3 and $-1/3$.

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A number of papers have recently appeared with estimates of the mass of the t quark.[1–3] A search is currently being made for $\bar{t}t$ bound states in e^+e^- annihilation.

In this paper we will analyze the mass relations for fermions in grand unified models, and as a result we will find an upper limit on the mass of the t quark.

We denote by $M(Q)$ $(Q = 2/3, -1/3, -1)$ the 3×3 mass matrices of the quarks (u, c, t) with charge 2/3, of the quarks (d, s, b) with charge $-1/3$, and of the leptons (e, μ, τ) with charge -1. In general, $M(Q)$ can be written[4]

$$M(Q) = \sum_a F^a(Q) G^a, \tag{1}$$

where the $F^a(Q)$ are numbers, and the G^a are 3×3 matrices which are independent of Q. Glashow[4] has pointed out a possibility that arises when the sum in (1) contains only two terms; such a situation arises, for example, in the SO(10) grand unification model, where only a single complex Higgs field contributes to the mass of each of the fermions in the **10** and **126** representations, and in the E_6 model, if **27** and **251** are chosen as the Higgs fields. From (1) we then find

$$M(-1) = \alpha M(2/3) + \beta M(-1/3). \tag{2}$$

We assume that the mass matrices are symmetric (as they are in the examples just cited). We can then rewrite (2) for the diagonalized mass matrices:

$$AM^0(-1)A^T = \alpha M^0(2/3) + \beta K^+ M^0(-1/3) K^*, \tag{3}$$

where the $M^0(Q)$ are diagonal (but generally complex) matrices, and A and K are unitary matrices. The matrix K describes the mixing in weak left-hand currents.[5] Within a redefinition of the phases, the matrix K can be expressed in terms of the three mixing angles θ_1, θ_2, θ_3 and the single phase δ, which is associated with CP violation[5] (the angle θ_1 is the same as the Cabibbo angle θ_C). If we ignore the mixing angles θ_1, θ_2, θ_3 ($\sin\theta_1 = 0.23$; see Refs. 6 and 7 regarding limitations on the quantities θ_2 and θ_3), we find that the matrix K becomes diagonal. From (3) we then find a relation for the masses of the particles[1)]:

$$\det \begin{pmatrix} ue^{i\phi_u} & ce^{i\phi_c} & te^{i\phi_t} \\ de^{i\phi_d} & se^{i\phi_s} & be^{i\phi_b} \\ ee^{i\phi_e} & \mu e^{i\phi_\mu} & \tau e^{i\phi_\tau} \end{pmatrix} = 0\,, \tag{4}$$

where the symbols for the particles represent the masses of the particles, which appear in (4) with arbitrary phases (these phases were set equal to zero in Ref. 4). From (4) we find

$$t \leqslant \frac{\tau(cd+us) + b(u\mu + ec)}{d\mu - es}\,. \tag{5}$$

The current masses of the quarks in (5) are functions of the momentum transfer. Following Ref. 4, we define the "observable" mass of the heavy quark, q, at a point as equal to the mass of the $\bar{q}q$ bound state (the masses of the light u, d, and s quarks are determined for a momentum transfer of 1 GeV). Taking the values $m(u) = 4.2$ MeV, $m(d) = 7.5$ MeV, $m(s) = 150$ MeV, $m(c) = 1.2$ GeV, and $m(b) = 4.4$ GeV (Ref. 1), we find the following limitation on the mass of the t quark:

$$\begin{aligned} m(t) &\leqslant 24.0 \text{ GeV for } \Lambda = 0.1 \text{ GeV} \\ m(t) &\leqslant 22.5 \text{ GeV for } \Lambda = 0.3 \text{ GeV}, \end{aligned} \tag{6}$$

where Λ is the well-known scale parameter of quantum chromodynamics.

How could the incorporation of mixing angles alter this result? To answer this question, we take the following approach. We assume that the mass matrices of the fermions are of the form

$$M(Q) = \begin{pmatrix} 0 & a(Q) & 0 \\ a(Q) & 0 & b(Q) \\ 0 & b(Q) & c(Q) \end{pmatrix}, \tag{7}$$

which leads to plausible values for the mixing angles.[1] For simplicity, we assume that the matrices in (7) are real; i.e., we ignore the *CP* violation. The elements of matrix (7) are then expressed in terms of the masses of the particles in the following manner[1]:

$$a^2(2/3) \approx uc, \qquad b^2(2/3) \approx ct, \qquad c^2(2/3) \approx (t-c)^2 \tag{8}$$

(with corresponding expressions for $Q = -1/3, -1$). For the Cabibbo angle we find[1,8]

$$\sin\theta_c \approx \sqrt{\frac{d}{s}} \pm \sqrt{\frac{u}{c}}\,, \tag{9}$$

where the $\pm$ correspond to the values $\delta = 0$ and $\delta = \pi$ of the *CP*-violation phase δ). Working from (2), (7), and (8), and using (9) (Ref. 8), we find a limitation on the mass of the t quark [the maximum value of $m(t)$ corresponds to $\delta = \pi$]:

$$m(t) \leqslant 24.5 \text{ GeV for } \Lambda = 0.1 \text{ GeV}$$
$$m(t) \leqslant 24.3 \text{ GeV for } \Lambda = 0.3 \text{ GeV} \tag{10}$$

not very different from the limitations in (6). This result remains in force when *CP* violation is incorporated. If $\sin\delta$ is small, then the upper limit on $m(t)$ does not change in first order in $\sin\delta$.

It can thus be expected that a $\bar{t}t$ bound state will have a mass less than 50 GeV.

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[1)]Here we have eliminated the nonphysical solutions of (3) of the type $\tau e^{i\phi_\tau} = \alpha u e^{i\phi_u} + \beta d e^{i\phi_d}, \mu e^{i\phi_\mu} = \alpha c e^{i\phi_c} + \beta s e^{i\phi_s}$, and $e e^{i\phi_e} = \alpha t e^{i\phi_t} + \beta b e^{i\phi_b}$, where (for example) the mass of the heavy τ lepton is coupled with the masses of the light u and d quarks.

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