Radially excited pseudoscalar states and the problem of the ι(1440) meson

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A model of strong mixing in a radially excited pseudoscalar nonet is proposed by analogy with mixing in the ground nonet. Comparison of the predicted mass formulas and the relations between the probabilities for the production of isoscalar particles with experimental data on the resonances $\pi'(1205)$, $\zeta(1275)$, and $\iota(1440)$ indicates a possible mixing of these states in a single radial nonet.

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The recently discovered pseudoscalar isoscalar resonance[1] $\iota(1440)$ can be interpreted as a gluon or even as a radially excited state[2] η'_R in a single SU_3^f nonet P_R with the resonances $\pi'(1205) = \pi_R$ (Ref. 3), $\zeta(1275) = \eta_R$ (Ref. 4), and, possibly, $K'(\sim 1400) = K_R$ (Ref. 5). Assuming that $\eta_R - \eta'_R$ mixing can be described by analogy with the $\eta - \eta'$ mixing in the pseudoscalar nonet $P = (\pi, \eta, \eta', K)$, we adopt a natural model for the $\eta_R - \eta'_R$ mixing, and we find that this approach allows us to identify $\iota(1440)$ with η'_R.

A basic property which distinguishes the mixing in a P nonet from other nonets is the maximum splitting of the masses of the π and η mesons.[6] Since the mixing parameter ϵ_P^2 depends strongly on the masses of the particles (in other words, on the effective masses of the quarks, which vary from particle to particle), this condition of extreme splitting cannot be expressed as a simple relationship between masses. Furthermore, the $q\bar{q}$ components in η and η' are noticeably nonorthogonal. If we ignore the nonorthogonality, we find the prediction $\theta_P \cong -\arctan(1/2\sqrt{2}) \cong -19.47°$ for the singlet-octet mixing angle θ_P, so that the mixing angle for strange and nonstrange quarks, $\theta_{\eta\eta'} \equiv \theta_P - \theta_0 + 90°$, should be $\theta_{\eta\eta'} \cong \theta_0 \equiv \arctan(1/\sqrt{2}) \cong 35.26°$. Using this mixing mechanism, we found correct masses for η' and K, and the mixing angle agrees well with data on η and η' production [here and below, we are using the notation $0.532(48) = 0.532 \pm 0.048$, etc.]:

$$K_{\eta\eta'} \equiv \frac{\sigma(\pi^- p \to \eta' n)}{\sigma(\pi^- p \to \eta n)} = 0.532\ (48),\ R_{\eta\eta'} \equiv \frac{B(J/\Psi \to \gamma\eta')}{B(J/\Psi \to \gamma\eta)} = 5.88\ (1.46).^{7} \qquad (1)$$

Here we have given a weighted average of $K_{\eta\eta'}$ over three experiments with a large statistical base.[8] The value found for $R_{\eta\eta'}$ in Ref. 7 does not contradict earlier measurements, which give a slightly smaller $R_{\eta\eta'}$, but the effect amounted to no more than 2σ because of the poor η' statistics. Determining the angles from $K_{\eta\eta'} = \tan^2\theta_{\eta\eta'}$ and $R_{\eta\eta'} = (k_{\eta'}/k_\eta)^3\cot^2\theta_P$, we find $\theta_{\eta\eta'} = 36.0(1.3)^\circ$ and $\theta_P = -20.8(2.4)^\circ$, [i.e., $\theta_{\eta\eta'} = 33.9(2.4)^\circ$]. It was shown in Ref. 9 that the predicted value of θ_P agrees with data on the radiative decays $V \to P\gamma$, $P \to V\gamma$, and $P \to \gamma\gamma$ of vector mesons, $V = (\rho, \omega, \varphi, K_V)$ and of pseudoscalar mesons, except $\Gamma(\eta \to \gamma\gamma)$. Using new data on these decays, we can show that the best value of $\theta_{\eta\eta'}$ is $\theta_{\eta\eta'} = 35.8(1.8)^\circ$, i.e., $\theta_P = -18.9(1.8)^\circ$[$\Gamma(\eta \to \gamma\gamma)$ was not used in the fit]. The weighted average of all these determinations is

$$\bar{\theta}_{\eta\eta'} = 35.6(1.0)^\circ, \theta_P = -19.1(1.0)^\circ.$$

Putting aside the possibility that this apparent agreement of very independent measurements of $\theta_{\eta\eta'}$ with each other and with the theoretical prediction of Ref. 6 is simply a fortuitous result, we may conclude that the experiment convincingly supports the mixing mechanism proposed in Ref. 6. As was mentioned in Ref. 9, only the predicted width of the η meson, $\Gamma(\eta \to \gamma\gamma) = 0.65$–$0.75$ keV, is in disagreement with both the value 0.323(46) keV adopted in Ref. 5, found in a single experiment using the Primakoff effect, and the result of an earlier experiment, 1.00(22) keV, based on the same method. It is obvious that $\Gamma(\eta \to \gamma\gamma)$ must be measured by a fundamentally different method, e.g., the $\gamma\gamma$ production of η in e^+e^- annihilation.

The layout of the P_R nonet can now be understood by analogy with P. We will begin by roughly estimating the mass π_R, using the empirical relation[6] $a'_{ij}(M^2_{ij,R} - M^2_{ij}) = \beta$ where a'_{ij} is the slope of the Regge trajectory for the states $q_i\bar{q}_j$, and M_{ij} and $M_{ij,R}$ are the masses of the ground state and the radially excited state, respectively, of $q_i\bar{q}_j$. Using the value[6] $a'_{c\bar{c}} \cong 0.346$, we find $\beta \cong 1.36$. From the relation $\pi_R^2 = \pi^2 + \beta/a'_{u\bar{u}}$, where $a'_{u\bar{u}} \cong 0.91$ (Ref. 6), it follows that $\pi_R \cong 1.23$ GeV. Noting that β may depend slightly on i and j and that the slope $a'_{u\bar{u}}$ is determined within $\sim 5\%$, we can find a cruder but more reliable estimate: $\pi_R = 1.20$–1.25. This estimate is consistent with the experimental value of Ref. 3. The expressions for the masses of the particles of the P_R nonet can be written[6]

$$\pi_R^2 = m_R^2 - 2\Delta_R^2,\ K_R^2 = m_R^2 - \frac{\Delta_R^4}{K_R^2},$$
$$\eta_R^2 = m_R^2 + 6\epsilon_R^2 - 2\delta_R^2,\ \eta_R'^2 = \eta_R^2 + 4\delta_R^2, \qquad (2)$$

where m_R^2, Δ_R^2, ϵ_R^2, and δ_R^2 are parameters:

$$\Delta_R^2 - \epsilon_R^2 = \delta_R^2 \cos(2\theta^R_{\eta\eta'}),\quad 2\sqrt{2}\,\epsilon_R^2 = \delta_R^2 \sin(2\theta^R_{\eta\eta'})\ ; \qquad (3)$$

and $\theta^R_{\eta\eta'}$ is a mixing angle, analogous to $\theta_{\eta\eta'}$. We will set $\theta_{\eta\eta'} \equiv \theta$. Also,[6] $\Delta_R^2 \equiv s_R^2 - u_R^2 \geqslant s^2 - u^2 \equiv \Delta^2 \cong 0.109$ GeV^2. We expect $0 < \epsilon_R^2 \lesssim \epsilon_P^2$ and, correspondingly,

$\theta \lesssim \theta_{\eta\eta'}$. Examining the dependence of π_R, η_R, and η'_R on m_R, δ_R, and $\theta^R_{\eta\eta'} \equiv \theta$, we see that the Jacobian $\partial(\pi_R, \eta_R, \eta'_R)/\partial(m_R, \delta_R, \theta)$ vanishes if $\tan(2\theta) = \sqrt{2}$, i.e., $\theta = 1/2(90 - \theta_0)^\circ \equiv \theta_1 \cong 27.4^\circ$; in this case, $\theta^R_P \equiv \theta + \theta_0 - 90^\circ = -\theta_1 \cong -27.4^\circ$. If the masses in P_R are such that $\theta \sim \theta_1$, then the exact value of θ cannot be determined from the masses, and the masses themselves are close to the extreme values. The exact meaning of this assertion can be seen from the mass formula

$$\eta'^2_R + \eta^2_R - 2\pi^2_R = \sqrt{3}(\eta'^2_R - \eta^2_R)\sin(2\theta + \theta_0), \tag{4}$$

which is easily derived with the help of (2) and (3). It follows immediately from (4) that, with fixed π_R and η'_R, the mass η_R reaches its maximum value if $\theta = \theta_1$. In a corresponding sense, the masses π_R and η'_R are minimal. These extreme values of the masses satisfy

$$(\sqrt{3} - 1)\eta'^2_R + 2\pi^2_R = (\sqrt{3} + 1)\eta^2_R , \tag{5}$$

which is satisfied strikingly well for

$$\pi_R = 1.205(7)^3, \ \eta_R = \zeta = 1.275(15)^4, \ \eta'_R = \iota = 1440(15)^1.$$

Relation (5) can be used to predict one mass from two others. For example, from the masses in (6) we find the predictions

$$\zeta = 1.272(8), \quad \pi_R = 1.209(23), \quad \iota = 1449(55).$$

The rather low value $K_R = 1.285(5)$ is predicted for K_R. Although this prediction does not agree with the estimated mass of the $K\pi\pi$ resonance, $K'(\sim 1400$; Ref. 5), it should be taken into account that this resonance is very wide, and the estimate of the mass is very rough. We note that with $|\theta - \theta_1| \lesssim 10^\circ$ Eq. (4) is essentially the same as (5), and in this interval of θ the predicted masses are essentially independent of θ.

In contrast, the parameters $K_{\iota\zeta}$ and $R_{\iota\zeta}$, determined by analogy with (1), depend very strongly on θ. With $|\theta - \theta_1| \lesssim 5^\circ$ we can estimate them by using the linear approximation: $K_{\iota\zeta} \cong 0.27 + (\theta - \theta_1)^\circ/40^\circ$ and $R^{-1}_{\iota\zeta} \cong 0.32 - (\theta - \theta_1)^\circ/40^\circ$. So far, we have no reliable data on these quantities, but we can extract some rough estimates out of data on ι and ζ production: $R^{-1}_{\iota\zeta} \lesssim 0.25$ and $K_{\iota\zeta} \lesssim 0.4$ (Refs. 1, 2, 4). We would thus have $30^\circ \lesssim \theta \lesssim 32^\circ$. With $25^\circ \lesssim \theta \lesssim 35^\circ$, the inequality $0.58 \lesssim R^{-1}_{\iota\zeta} + K^{-1}_{\iota\zeta} \lesssim 0.65$ should hold. A substantial deviation from this inequality would unambiguously mean that the simple interpretation proposed for $\iota(1440)$ must be ruled out. We do not, however, rule out the possibility that $\iota(1440)$ contains a mixture of η'_R and gluonium. In this case, the estimates above are no longer in force, and it is not a simple matter to decipher the layout of ι. Another possibility is a significant mixing of η' and η'_R. Our quantitative estimates would then have to be modified slightly, but there would be no qualitative change in the overall situation.

The possibility that ι and ζ occur in a P_R nonet was first discussed by Chanowitz,[10] but the mass formulas and quantitative mixing models were not taken up, and some information on π_R, which appeared later,[3] was not included. Working from some qualitative estimates of the parameters $K_{\iota\zeta}$ and $R^{-1}_{\iota\zeta}$, Chanowitz concluded that it was unlikely that $\iota(1440)$ would be identified with η'_R. The arguments of the present paper show that this conclusion cannot be considered solid. A final resolution of the true nature of $\iota(1440)$ will require more accurate estimates of $K_{\iota\zeta}$ and $R_{\iota\zeta}^{-1}$. It would also be desirable to have some more comprehensive data on the properties (and existence) of the resonances π_R, ζ, and K', necessary for a reliable quantitative description of the P_R nonet.

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