# Spontaneous breaking of conformal symmetry; anomalous dimensionalities for instantons and merons

M. Ya. Pal'chik and E. S. Fradkin
*P.N. Lebedev Physics Institute, Academy of Sciences of the USSR, Moscow*

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The role of quantum corrections to the instanton and meron solutions is discussed under the assumption that these corrections do not alter the symmetry of the solutions. Exact expressions are derived for the average field in the scalar theory. The equations for the constant parameters in these expressions are discussed.

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In this letter we wish to show that the sum of all the radiative corrections to the instanton[1] and meron[2] solutions can be found by the methods of conformal field theory (see Refs. 3 and 4, for example). It turns out that the entire effect of the radiative corrections is to give rise to anomalous dimensionalities. To illustrate the method we will examine the theory for a scalar field. Gauge theories will be considered in a separate paper.

The instanton and meron solutions result from a spontaneous breaking of conformal symmetry.[5,2] The residual symmetry in the case of the meron solutions is the maximum compact subgroup SO(4)×SO(2) of the conformal group, while in the case of the instanton solutions (in a Euclidean space) the residual symmetry is the SO(5) group, which is a subgroup of the SO(5,1) group of the Euclidean space. This approach is based on the assumption that this symmetry of the classical solution is retained after the radiative corrections are summed.

We denote by $\phi(x)$ a conformal field with a scale dimensionality $d$:

$$[\phi(x), P_\mu] = i\partial_\mu\phi(x), \qquad [\phi(x), M_{\mu\nu}] = i(x_\mu\partial_\nu - x_\nu\partial_\mu)\phi(x),$$
$$[\phi(x), D] = i(x_\mu\partial_\mu + d)\phi(x), \qquad [\phi(x), K_\mu] = i[-x^2\partial_\mu + x_\mu(x_\nu\partial_\nu + d)]\phi(x). \tag{1}$$

In the case of a spontaneously broken symmetry, the average field can be found[5] from the first-order differential equations which express the symmetry of the vacuum. We denote by $\Omega_1$ the SO(4)×SO(2)-invariant vacuum corresponding to a meron solution:

$$\mathbf{M}|\Omega_1> = \mathbf{S}|\Omega_1> = R_0|\Omega_1> = 0, \tag{2}$$

where $M$ and $S$ are the generators of the group SO(4)∼SO(3)×SO(3), and $R_0$ is the generator of SO(2):

$$\mathbf{M} = (M_{23}, M_{31}, M_{12}), \quad \mathbf{S} = 1/2\left(a\mathbf{P} - \frac{1}{a}\mathbf{K}\right), \quad R_0 = 1/2\left(aP_0 + \frac{1}{a}K_0\right), \tag{3}$$

where $a$ is a parameter with the dimensionality of a length. From (1)–(3) we find ($i = 1, 2, 3$)

$$\left(\frac{a^2 + x^2}{2}\partial_i - x_i x_\mu\partial_\mu - x_i d\right)B_1(x) = 0, \quad (x_i\partial_k - x_k\partial_i)B_1(x) = 0,$$
$$\left(\frac{a^2 - x^2}{2}\partial_0 + x_0 x_\mu\partial_\mu + x_0 d\right)B_1(x) = 0.$$

The most general solution of these equations is (for $a = 1$)

$$B_1(x) = <\Omega_1|\phi(x)|\Omega_1> = \frac{b_1}{[(1 + t_+^2)(1 + t_-^2)]^{d/2}}, \tag{4}$$

where $b_1$ is a constant, and $t_\pm = x_0 \pm |\mathbf{x}_1|$.

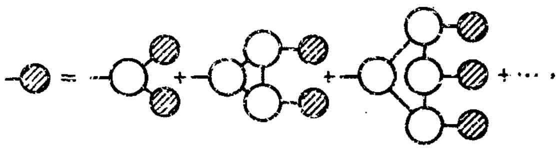
Let us consider the instanton solution. The SO(5) group has ten generators: $M_{\mu\nu}$ and $R_\mu = (1/2)(aP_\mu + \frac{1}{a}K_\mu)$. Introducing the SO(5)-symmetric vacumm $|\Omega_2\rangle, M_{\mu\nu}|\Omega_2\rangle = R_\mu|\Omega_2\rangle = 0$, we find

$$B_2(x) = <\Omega_2|\phi(x)|\Omega_2> \frac{b_2}{(a^2 + x^2)^d}. \tag{5}$$

It is important to note that (4) and (5) are the most general expressions compatible with the symmetry of the vacuum. The dimensionality of $d$ can take on any real values $d \geqslant 1$ (positive). In the limit of the canonical value $d = 1$, expressions (4) and (5) become the ordinary classical instanton and meron solutions.[2,5] The role of the radiative corrections thus reduces to one of changing the dimensionality from the canonical value $d = 1$ to some anomalous value $d > 1$.

To calculate this value we note that the scale dimensionality is a measure of the transformation properties of the field $\phi(x)$ after conformal transformations. As a characteristic of the field operator, it does not depend on the properties of the less symmetric vacuums $\Omega_1$ and $\Omega_2$. To calculate it, it is thus sufficient to consider the sector associated with the conformally invariant vacuum (which is not necessarily stable). The dimensionality $d$ can thus be calculated from the ordinary bootstrap program[6] of the conformal theory (see also Refs. 3 and 4; the corresponding solution for the third- and fourth-order interaction series was derived in Refs. 3 and 7).

The constants $b_1$ and $b_2$ can be calculated from the skeletal equations for the average field. As an example we consider the simplest ternary interaction,



(6)

where a hatched vertex denotes the average field in (4) or (5), while the unhatched vertices and the interior lines are associated with the conformal vertices and propagators, whose exact form is known.[8] Instead of (6), we can use a closed form of the equations in which the summation is carried out over the internal field. We will consider that approach in a separate paper. It is important to note that the symmetry of each part in (6) is the same as the symmetry of the average field, since the internal integrations are conformally invariant. As a result, the coordinate dependence of each part is the same as on the left side. Canceling out this dependence, we find an algebraic equation for $b_1$ and $b_2$.

An interesting application of this approach can be made in theories in which the condensate is described by quadratic field combinations. We have verified that in the Tirring model an exact solution of the meron type exists and can be found; for this solution, we have $\langle\Omega_2| j_\mu(x)|\Omega_2\rangle \neq 0$, but $\langle\Omega_2|\psi(x)|\Omega_2\rangle = 0$, where $j_\mu(x) = \bar{\psi}\gamma_\mu\psi$ is a conserved current. Similar solutions are possible in electrodynamics.

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