Vacuum polarization effects in the mesic molecules $dd\mu$ and $dt\mu$

V. S. Melezhik
Joint Institute for Nuclear Research

(Submitted 22 June 1982)
Pis'ma Zh. Eksp. Teor. Fiz. **36**, No. 3, 101–103 (5 August 1982)

The shifts $\Delta\epsilon_{11}^{V.p.}$ of the energy $\epsilon_{J\text{v}}$ of the weakly bound states ($J = \text{v} = 1$) of the mesic molecules $dd\mu$ and $dt\mu$, which are caused by polarization of the electron-positron vacuum, are calculated. The results are $\Delta\epsilon_{11}^{V.p.}(dd\mu) = 10$ meV and $\Delta\epsilon_{11}^{V.p.}(dt\mu) = 6.5$ meV. The calculations are carried out in the adiabatic representation of the three-body problem.

PACS numbers: 36.10.Dr, 31.30.Jv, 12.20.Ds

1. The resonance production of the mesic molecules $dd\mu$ and $dt\mu$ has recently attracted much interest,[1] for at least two reasons: in connection with the possible use of this production mechanism for muon catalysis of nuclear fusion reactions[2] and for spectroscopic measurements. A quantitative description of the kinetics of μ catalysis requires knowledge of the binding energy of the rotational-vibrational states ($J = \text{v} = 1$) which are produced in resonance reactions of mesic molecules, and these binding energies must be known highly accurately, within ~2–3 meV, i.e., within an uncertainty smaller than the corrections (~50 meV)[1)] to the Coulomb energies (for relativistic effects, for the electromagnetic structure of the nuclei, and for electron screening[2]). In this letter we wish to examine the effect of the polarization of the electron-positron vacuum, which is known to be the predominant radiative correction[4] to the Coulomb interaction of the μ meson with nuclei.

This calculation may also be of interest in connection with the possibility of direct measurements of the polarization shift $\Delta\epsilon_{11}^{V.p.}(dd\mu)$, since experimental apparatus is capable of finding the resonance energy of the reaction $d\mu + d \xrightarrow{\lambda dd\mu} dd\mu$ very accurately, within $\sim 10^{-2}$ meV (Ref. 5).

2. The potential describing the screening of the charges in a mesic molecule due to the vacuum polarization is the sum

$$V(\mathbf{r}, \mathbf{R}) = V(|\mathbf{r}_a|) + V(|\mathbf{r}_b|) - V(|\mathbf{R}|) \tag{1}$$

of binary Juling potentials[4,6]

$$V(|\mathbf{r}_{a,b}|) = -\frac{2a}{3\pi}\frac{1}{|\mathbf{r}_{a,b}|}\int_1^\infty \sqrt{x^2-1}\left(1+\frac{1}{2x^2}\right)e^{-2\gamma x|\mathbf{r}_{a,b}|}\frac{dx}{x^2}, \tag{2}$$

which reflect only the single-loop diagrams in the gamma propagator of oppositely charged point particles (the meson and the nucleus), and the potential $V(|\mathbf{R}|)$, which describes the distortion of the Coulomb interactions of the nuclei. Here $|\mathbf{r}_{a,b}| = |\mathbf{r} \pm (1/2)R|$ are the distances from the μ meson to the nuclei, R is the distance between the nuclei, r is the distance from the center of the charges of the nuclei to the μ meson, $\gamma = m_e/a$, and m_e is the mass of the electron. We are using the system of units with $\hbar = e^2 = m_a = 1$, where $m_a^{-1} = m_\mu^{-1} + M_t^{-1}$, m_μ is the mass of the μ meson, and M_t is the mass of the triton (in the case of $dd\mu$, we have $m_a^{-1} = m_\mu^{-1} + M_d^{-1}$). We are ignoring double-loop diagrams (the Källén-Sabry potential) and also some other factors, which make corrections to the shift $\Delta\epsilon_{11}^{V.p.}$ which are no larger than those due to the double-loop diagrams: the fact that the nuclei are not point particles and the polarization of the vacuum due to $\mu^+\mu^-$ pairs and other particles.[6]

We can find the correction[1)] $\Delta\epsilon_{J\nu}^{V.p.}$ to the Coulomb binding energy $\epsilon_{J\nu}$ of the $(J\nu)$ state of the mesic molecule in first-order perturbation theory in $V(\mathbf{r}, \mathbf{R})$:

$$\Delta\epsilon_{J\nu}^{V.p.} = \int\int \Psi_{J\nu}^2(\mathbf{r}, \mathbf{R})\, V(\mathbf{r}, \mathbf{R})\, d^3r\, d^3R - \Delta E_{1s}^{V.p.}. \tag{3}$$

As the zeroth-approximation wave functions $\Psi_{J\nu}(\mathbf{r}, \mathbf{R})$ we used the eigenfunctions of the Coulomb Hamiltonian of the mesic molecule, calculated in Ref. 3 in the adiabatic representation of the three-body problem. The distortion of the function by relativistic effects is slight and is ignored here. The shift of the energy level, $\epsilon_{J\nu}$, is then equal to the sum

$$\Delta\epsilon_{J\nu}^{V.p.} = \sum_{i,j} \Delta E_{ij}^{J\nu} \tag{4a}$$

of the contributions of the pairs of states (i,j) of the two-center problem,

$$\Delta E_{ij}^{J\nu} = \int_0^\infty \chi_i^{J\nu}(R)\,(V_{ij}(R) - \Delta E_{1s}^{V.p.}\,\delta_{ij})\,\chi_j^{J\nu}(R)\, dR, \tag{4b}$$

where

$$V_{ij}(R) = \int \varphi_i(\mathbf{r}; \mathbf{R})\, V(\mathbf{r}, \mathbf{R})\, \varphi_j(\mathbf{r}; \mathbf{R})\, d^3r \tag{4c}$$

are the matrix elements of operator (1) in the adiabatic representation. Here $\varphi_j(\mathbf{r};\mathbf{R})$ are the wave functions of the two-center problem, which are ordinarily designated by the set of parabolic, $[n_1n_2m]$, or spherical, (Nlm), quantum numbers and are characterized by a parity $p = (g, u) = (-1)^l$ with respect to the inversion[3] $\mathbf{r} \to -\mathbf{r}j = (jp) = [n_1n_2mp] = (Nlm)$ [$\chi_j^{J\nu}(R)$ are the wave functions of the relative motion of the nuclei in the mesic molecule]. In order to calculate the matrix elements $V_{ij}(R)$, it is convenient to single out in the original operator, (1),

$$V(\mathbf{r}, \mathbf{R}) = V^{(-)}(\mathbf{r}, \mathbf{R}) + V^{(+)}(|\mathbf{R}|) \tag{5a}$$

the part $V^{(+)}(|\mathbf{R}|)$ which is diagonal in the adiabatic basis:

$$V^{(-)}(\mathbf{r}, \mathbf{R}) = V\left(\left|\mathbf{r} + \frac{1}{2}\mathbf{R}\right|\right) + V\left(\left|\mathbf{r} + \frac{1}{2}\mathbf{R}\right|\right), V^{(+)}(|\mathbf{R}|) = -V(|\mathbf{R}|). \tag{5b}$$

The matrix elements $V_{ij}^{(-)}(R) = V_{ji}^{(-)}(R) = V_{[n_1n_2],[n_1'n_2']}^{(-)}(R)\delta_{mm'}\delta_{pp'}$ were calculated

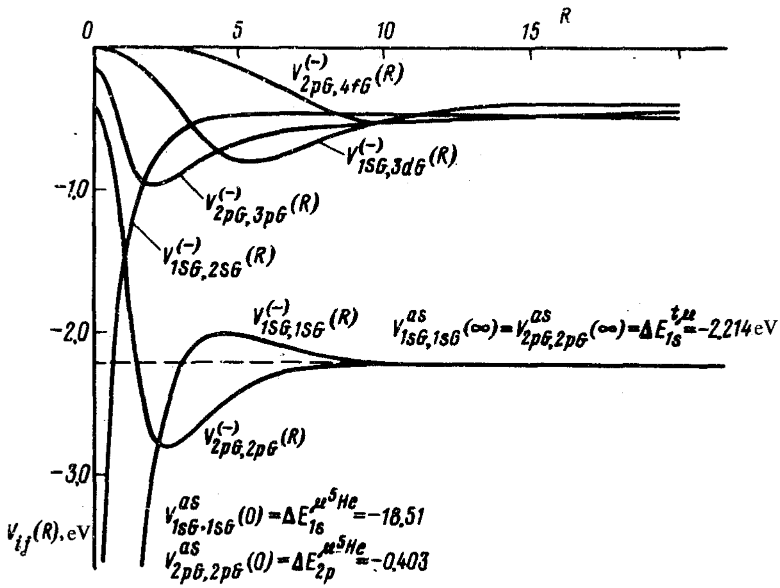

FIG. 1. The matrix elements $V^{(-)}_{ij}(R)$ for the $dt\mu$ system for the case $dd\mu\, V^{(-)}_{iu,ju}(R)\equiv 0$.

by the algorithm described in Ref. 7. Figure 1 shows the effective potentials $V^{(-)}_{ij}(R)$ for the first eight states $(i;j)$. The accuracy of the calculations was monitored by using the asymptotic relations

$$V^{(-)}_{ii}(R)\xrightarrow{R\to 0}\begin{cases}\Delta E^{V.p.}_{Nlm}(\mu^5\,\mathrm{He}),\ dt\mu\\ \Delta E^{V.p.}_{Nlm}(\mu^4\mathrm{He}),\ dd\mu\end{cases},$$
$$V^{(-)}_{ii}(R)\xrightarrow{R\to\infty}\begin{cases}\Delta E^{V.p.}_{Nlm}(\mu t),\ dt\mu\\ \Delta E^{V.p.}_{Nlm}(\mu d),\ dd\mu\end{cases},\qquad (6)$$

which were satisfied within a relative error $\sim 10^{-5}$ for the ground state $(i=1)$ and $\sim 10^{-3}$ for the excited states with $R=0$ and $R=20$; here $\Delta E^{V.p.}_{Nlm}$ are the polarization shifts of the levels of the mesic atoms.

3. The level shifts calculated for the mesic molecules $dt\mu$ and $dd\mu$ from (4) are $\Delta\epsilon^{V.p.}_{11}(dt\mu)=6.5$ meV and $\Delta\epsilon^{V.p.}_{11}(dd\mu)=10$ meV. The matrix elements $V^{(-)}_{kj}(R)$ which we have ignored are estimated to make contributions no larger than the assumed error of the calculations, $\sim 10^{-1}$ meV. We might note that the value found here for $\Delta\epsilon^{V.p.}_{11}(dd\mu)$ differs only slightly from the estimate in Ref. 8: $\Delta\epsilon^{V.p.}_{11}\sim 8$ meV (for $dt\mu$, $\Delta\epsilon^{V.p.}_{11}\sim -3$ meV). The near agreement results from a partial cancellation of the corrections $\Delta E_{ij}(i>1, j>1)$ calculated in the present study.

This effect makes a relatively large contribution to the Coulomb binding energy of the mesic molecule, $\sim 5\times 10^{-3}$–10^{-2} ($\sim 10^{-4}$ in mesic atoms[6]). At present, the factor restricting an experimental study of this effect is not the absence of reliable information on the nuclear form factors, in contrast with the mesic atoms,[6] $(\mu^4 \mathrm{He})^+_{2s}$, but the errors in the calculated Coulomb energies of the states of the mesic molecules and other relativistic corrections, which are comparable to the vacuum-polarization correction.

I wish to thank L. I. Ponomarev for interest in this study and for many discussions; D. Bakalov, S. I. Vinitskiĭ, and T. P. Puzynina for assistance; and V. G. Zinov for useful comments.

[1)] The Coulomb energies $\epsilon_{J\nu}$ of these states are[3] $\epsilon_{11}(dd\mu) = -1910$ meV and $\epsilon_{11}(dt\mu) = -640$ meV.
[2)] Here $\Delta E^{V.p.}_{1s}$ is the vacuum-polarization correction to the binding energy of the ground state of the mesic atom $\mu M_a (M_a \geqslant M_b)$, from which the energy of the states of the mesic molecule are customarily measured.[3]

[1]S. I. Vinitskiĭ *et al.*, Zh. Eksp. Teor. Fiz. **74**, 849 (1978) [Sov. Phys. JETP **47**, 444 (1979)].
[2]L. I. Ponomarev, Report to the Tenth European Conference on Controlled Thermonuclear Fusion and Plasma Physics, Moscow, 14–19 September 1981; Report R4-81-800, Joint Institute for Nuclear Research, Dubna, 1981.
[3]S. I. Vinitskiĭ *et al.*, Zh. Eksp. Teor. Fiz. **79**, 698 (1980) [Sov. Phys. JETP **52**, 353 (1980)].
[4]E. M. Lifshitz and L. P. Pitaevskiĭ, Relyativistskaya kvantovaya teoriya (Relativistic Quantum Theory), Part 2, Nauka, Moscow, 1971.
[5]L. N. Somov, Report R4-81-851, Joint Institute for Nuclear Research, Dubna, 1981.
[6]R. Borie and G. A. Rinker, Rev. Mod. Phys. **54**, 67 (1982).
[7]L. I. Ponomarev and T. P. Puzynina, Report R4 = 5040, Joint Institute for Nuclear Research, Dubna, 1970.
[8]V. S. Melezhik and L. I. Ponomarev, Phys. Lett. **77B**, 217 (1978).

Translated by Dave Parsons
Edited by S. J. Amoretty