Theory of stimulated Brillouin scattering in a low-density plasma

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The convective stimulated-Brillouin-scattering instability, which is usually considered in the case of an inhomogeneous low-density plasma, occurs only if the dissipation of sound is rapid. If this dissipation is only slight, an absolute stimulated-Brillouin-scattering instability occurs.

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Experiments on stimulated Brillouin (or "Mandel'shtam-Brillouin") scattering in an inhomogeneous plasma are usually interpreted in terms of a convective instability,[1,2] while an absolute stimulated-Brillouin-scattering instability occurs in a slab of a spatially homogeneous plasma.[3] In this paper we will resolve this contradiction of the theory by determining the range of applicability of the convective-instability arguments. We will see that an absolute instability can occur in a slab of a spatially inhomogeneous plasma, working in the particular case of stimulated Brillouin backscattering.

To analyze stimulated Brillouin backscattering in a nonisothermal ($T_e \gg T_i$), low-density, inhomogeneous plasma [$n_e(x) \ll n_c = m\omega_0^2/4\pi e^2$] we will use the geometric-optics approximation. We write the electromagnetic field as

$$\mathcal{E}_y = \mathrm{Re}\Big\{E_0\sqrt{k_0/k_0(x)}\exp\Big[i\int^x k_0(x')dx' - i\omega_0 t\Big] + E_1(x)\sqrt{k_1/k_1(x)}\exp\Big[-i\int^x k_1(x')dx' - i(\omega_0-\omega)t\Big]\Big\}.$$

Here ω_0 is the frequency, $k_0(x) = (\omega_0/c)\sqrt{1 - n_e(x)/n_c}$ and $k_0 = \omega_0/c$ are the wave vectors in the plasma and in vacuum, and E_0 is the given amplitude of the pump field

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incident on the plasma. The subscript 1 refers to the scattered wave. We are to derive the grain $K = E_1(-\infty)/E_1(+\infty)$. The beats of the incident and scattered waves resonant with the acoustic wave, with frequency ω and wave vector $k_s = \omega/v_s$, where v_s is the sound velocity. This interaction is strongest near the resonance points, where $k_0(x) + k_1(x) = k_s$. For a bell-shaped profile (Fig. 1), there are two such points: x_1 and x_2. Writing the density perturbation as $\delta n(x,t) = -in_e(x)[\nu_1(x)\exp(ik_s x - i\omega t) - \text{c.c.}]$, we find the following truncated equations for the amplitudes $E_1(x)$ and $\nu_1(x)$:

$$E_1' = -(1/2)\, k_0 a E_0 \nu_1 \exp(-i\Phi);$$

$$\nu_1'(x) + (\Gamma_s + a'/2a)\nu_1 = k_0 E_0 E_1^* (16\pi n_c \kappa_B T)^{-1} \exp(-i\Phi).$$

Here $\Phi(x) = \int^x \Delta k(x')dx'$, $\Gamma_s = \gamma_s/v_s$ is the spatial damping rate of the sound, κ_B is the Boltzmann constant, $\alpha(x) = [n_c/n_e(x) - 1]^{-1} \simeq n_e/n_c$, $\Delta k(x) = k_0(x) + k_1(x) - k_s \simeq k_0(\alpha_0 - \alpha(x))$, and $\alpha_0 = 2 - \omega/k_0 v_s$. Finally, for the function

$$s(x) = \nu_1^*(x) \exp\left[-(i/2)\int^x dx(\Delta k + i\Gamma_s/2 + ia'/4a\right]$$

the truncated equations yield $s'' + U(x)s = 0$, where

$$U(x) = \tfrac{1}{4}[k_0^2 I a(x) + (a'/a)' + 2\Gamma_s' - 2ik_0 a'(x)$$
$$+ k_0^2(a_0 - a - i\Gamma_s/k_0 - ia'/2k_0 a)^2],$$

and $I = |E_0|^2/8\pi n_c \kappa_B T$. The geometric-optics approximation holds over nearly the entire plasma volume:

$$s(x) = AV^{-1/4}\exp\left(i\int^x U^{1/2}dx\right) + BU^{-1/4}\exp\left(-i\int^x U^{1/2}dx\right).$$

It does not hold near the resonant points. To find the relationships among the coefficients A and B in regions I, II, and III (Fig. 1), we use a linear approximation of the plasma density profile near the resonant points: $\alpha(x) \simeq \alpha_0[1 + (x - x_{1.2})/L_{1.2}]$. This approach allows us to express $s(x)$ in terms of parabolic cylinder functions; then, working in the standard manner,[4] we can use the asymptotic expansions to derive the following relationships among the coefficients:

$$\begin{pmatrix} A_{II} e^{i\psi_1} \\ B_{II} e^{-i\psi_1} \end{pmatrix} = \hat{M}^+ \begin{pmatrix} A_I e^{i\psi_1} \\ B_I e^{-i\psi_1} \end{pmatrix}; \quad \begin{pmatrix} A_{III} e^{i\psi_2} \\ B_{III} e^{-i\psi_2} \end{pmatrix} = \hat{M}^- \begin{pmatrix} A_{II} e^{i\psi_2} \\ B_{II} e^{-i\psi_2} \end{pmatrix},$$

where the elements of the transition matrix $\hat{M}^\sigma$ ($\sigma = \text{sign}\, L_{1,2}$) are

$$M_{11}^\sigma = M_{22}^{-\sigma *} = \sqrt{2\pi}\,\Gamma^{-1}\left(i\kappa + \frac{1+\sigma}{2}\right)$$
$$\times \exp\left\{-\pi\kappa/2 + i\pi\sigma/4 + i(\kappa - i\sigma/2)[-1 + \ln(\kappa - i\sigma/2)]\right\},$$

$$M_{12}^\sigma = M_{21}^{-\sigma} = \sigma\exp(-\pi\kappa)$$

and

$$\psi_{1,2} = \int^{\tilde{x}_{1,2}} dx\, U^{1/2}(x), \quad \tilde{x}_{1,2} = x_{1,2} - IL_{1,2}/2a_0 - (i/2a_0k_0)(1 + 2\Gamma_s(x_{1,2})L_{1,2}).$$

In the complex x plane, the function $U^{1/2}(x)$ is determined by the condition $\mathrm{Re}U^{1/2}(x) > 0$. Here we have introduced the parameter

$$\kappa_{1,2} = (1/4)\, k_0 I\, |L_{1,2}| \left\{1 - (I/a_0) + (a_0 k_0^2 I)^{-1}[(a'_0/a_0)' + 2\Gamma'_s]\right\}.$$

We note that the transition matrix is derived under the assumptions

$$(k_0 a_0 a_0' / a_0'') \gg \max\left\{1,\ \Gamma_s(x_{1,2})|L_{1,2}|,\ |k_0 a_0 L_{1,2}|^{1/2},\ \kappa_{1,2}|k_0 a_0 L_{1,2}|^{1/2}\right\},$$

which correspond to a linear approximation of the density in the region in which the parabolic cylinder functions join with the geometric-optics approximation.

The electromagnetic field of the scattered wave in the limits $x \to \pm\infty$ is determined exclusively by the coefficients A. The coefficient A_{I} is a measure of the amplitude of the scattered wave which is amplified in the plasma, while A_{III} is the value of this amplitude at the entrance to the plasma. The coefficient B_{I} corresponds to the amplitude of the acoustic wave entering the plasma slab. If we assume that there is no source of sound, we have some definite boundary conditions which allow us to write the following general expression for the gain:

$$K = D^{-1} \exp\left\{2i(\psi_2 - \psi_1) - i\int_{-\infty}^{+\infty} dx\,[U^{1/2} - (\Delta k/2) + (i\Gamma_s/2) + i(a'/4a)]\right\},$$

where

$$D = M^-_{12} M^+_{21} + M^-_{11} M^+_{11} \exp[2i(\psi_2 - \psi_1)].$$

Let us analyze this result for the usual conditions $k_0^2\alpha_0^2 \gg k_0^2\alpha_0 I \gg \Gamma'_s$, $(\alpha'_0/\alpha_0)'$, and $\Gamma_s \ll \alpha_0 k_0$, when we have

$$K = D^{-1} \exp\left[(ik_0 I/4)\int_{-\infty}^{+\infty} dx\, a(x)/(a(x) - a_0)\right],$$

$$D = \exp[-\pi(\kappa_1 + \kappa_2)]\left\{1 + \exp\left[i\Psi - \int_{x_1}^{x_2} dx\,\Gamma_s(x)\right][\exp(2\pi\kappa_1) - 1]^{1/2}[\exp(2\pi\kappa_2) - 1]^{1/2}\right\},$$

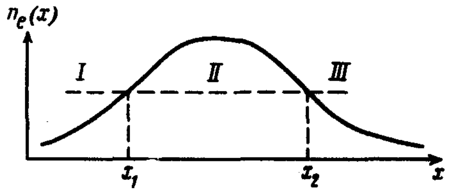

FIG. 1.

and the real phase is determined by

$$\Psi = \kappa_1 \ln|k_0 a_0 L_1| + \kappa_2 \ln|k_0 a_0 L_2| + k_0 \int_{x_1}^{x_2} dx[a - a_0 + {}^1/_2 aI/(a-a_0)]$$

$$- \arg[\Gamma(1 + i\kappa_1)\Gamma(1 + i\kappa_2)] + {}^{\pi}/_2 .$$

We see that the convective instability usually discussed, which corresponds to a gain $|K| \simeq \exp[\pi(\kappa_1 + \kappa_2)]$ and which occurs when $\pi\kappa_{1,2} \gg 1$, can occur only if the dissipation is extremely strong, such that $\pi(\kappa_1 + \kappa_2) < \int_{x_1}^{x_2} dx \Gamma_s(x)$. If, on the contrary, the absorption is slight, and the opposite inequality holds, the quantity $|K| \simeq \exp \int_{x_1}^{x_2} dx \Gamma_s(x)$ is independent of the pump intensity. This case corresponds to a saturation of the convective instability. If the dissipation of the sound is negligible, $\int_{x_1}^{x_2} dx \Gamma_s(x) \lesssim 1$, the convective instability does not occur at all.

The gain becomes infinite at $D = 0$; with $\mathrm{Im}\omega = 0$, this is the threshold condition for the absolute stimulated-Brillouin-scattering instability, while at $\mathrm{Im}\omega \neq 0$ it is the dispersion relation. Fuchs and Beaudry[5] have carried out a numerical study of the threshold for this instability in the case of a linear profile. In the present study we have the following simple and graphic expression for the threshold for the absolute stimulated-Brillouin instability ($\mathrm{Im}\Psi = 0$):

$$[\exp(\pi/2k_0 L_1 I_{\mathrm{thr}}) - 1]\,[\exp(\pi/2k_0|L_2|I_{\mathrm{thr}}) - 1] = \exp\left(2\int_{x_1}^{x_2} dx \Gamma_s(x)\right).$$

If $\pi k_0 |L_{1,2}| I_{\mathrm{thr}} \gg 1$, this expression means that the absolute stimulated-Brillouin instability corresponds to a cyclic motion (cf. Ref. 6) of waves which are trapped in the plasma, which are amplified near the resonant points, and which are damped as they propagate away from one resonant pair to the other (or, under other conditions, away from a resonant point to a reflection point).

Above the threshold the instability growth rate is

$$\gamma = \mathrm{Im}\,\omega = -\gamma_s + v_s |x_2 - x_1|^{-1} \ln[\exp \pi(\kappa_1 + \kappa_2) - 1].$$

A discrete spectrum of growing waves is excited with frequencies $\omega_n \approx 2k_0 v_s$; adjacent frequencies are separated by $|\omega_{n+1} - \omega_n| \approx 2\pi\omega_n / k_0 |x_2 - x_1|$. Under the natural assumption that $x_2 - x_1$ is on the order of $10\,\mu$m, this discrete spectrum may correspond to that observed in experiments on the effect of the beam from an Nd laser on the corona of the stimulated-Brillouin-scattering spectrum, which is wavelength-modulated with a period ~ 1 Å (Ref. 7).

According to our theory, the threshold energy flux density of the electromagnetic radiation is $q_{\mathrm{thr}} \approx c n_c \kappa_B T(\gamma_s/\omega_s)$; this expression yields values in approximate agreement with the observed values.

In summary, we have theoretically demonstrated the possibility of an absolute stimulated-Brillouin instability in a spatially inhomogeneous plasma. The conditions for the occurrence of this instability have been shown to be much broader than the conditions for the occurrence of the convective stimulated-Brillouin instability, which has been widely discussed.

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