Can a complicated vacuum structure cause a breaking of supersymmetry?

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The incorporation of a complicated structure of the vacuum in non-Abelian gauge theories does not cause a breaking of supersymmetry.

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There has recently been a revival of interest in the supersymmetry models of field theory[1] because it is hoped that the problem of the "hierarchy of scales" can be solved by working in supersymmetry models.[2] The supersymmetry must be broken in order to obtain the observed mass spectrum. Belavin *et al.*[2] have discussed the possibility that instanton effects would break the supersymmetry (i.e., the possibility that the supersymmetry would be broken as a result of the incorporation of a complicated vacuum structure in the gauge theories[3–6].

We will show in this paper that the incorporation of a complicated vacuum structure in the supersymmetry gauge theories cannot by itself cause a breaking of the supersymmetry. For definiteness, we consider the simplest supersymmetry gluon dynamics. The Lagrangian of the model in the Vass–Zumino gauge is[1]

$$\mathcal{L}=-\frac{1}{4}(F^{a}_{\mu\nu})^{2}+\frac{i}{2}\bar{\psi}^{a}D_{ab}\psi^{b}+\frac{1}{2}(D^{a})^{2}, \tag{1}$$

$$\hat{D}_{ab}=\partial_{\mu}j^{\mu}\delta^{a}_{b}-igA^{k}_{\mu}j^{\mu}T^{k}_{ab}.$$

Here ψ^{a} is the Majorana field, a supersymmetry partner of the gauge field A^{a}_{μ}, and T^{k}_{ab} are the matrices of the associated representation of the algebra of the gauge group. The conserved supersymmetry current corresponding to the Lagrangian in (1) is

$$J^{s}_{\mu}(x)=\frac{1}{2}F^{a}_{\alpha\beta}(x)\sigma_{\alpha\beta}j_{\mu}\psi^{a}(x). \tag{2}$$

The supersymmetry current in (2) is gauge-invariant, and it remains so when quantum corrections[7] are incorporated in a perturbation theory. We will assume here that even outside perturbation theory the supersymmetry current $J^{s}_{\mu}(x)$ or, more precisely, the generator of the supersymmetry, $Q_{s}=\int J^{S}_{0}(x)d^{3}x$, is gauge-invariant, i.e,

$$[Q_{s},u]=0, \tag{3}$$

where u represents the gauge-transformation operators, including those which are topologically nontrivial. Actually, the main conclusion of this study—that the incorporation of a complicated vacuum structure does not in itself cause a breaking of the supersymmetry—follows from commutation relation (3). The θ vacuum can be written as[6]

$$|\theta\rangle=\sum_{n=-\infty}^{n=\infty}e^{in\theta}[u(1)]^{n}|0\rangle. \tag{4}$$

Here $|0\rangle$ is the vacuum of the perturbation theory, and $u(1)$ is a topologically nontrivial gauge transformation with a change in topological number $\Delta n=1$. In the gauge $A^{0}_{a}=0$, we have

$$\Delta n=\frac{1}{6\pi^{2}}\int\epsilon^{ijk}f^{abc}A^{i}_{a}A^{j}_{b}A^{k}_{c}d^{3}x.$$

Let us assume that the state $|0\rangle$ is supersymmetry-invariant, i.e.,

$$Q_{s}|0\rangle=0. \tag{5}$$

It follows immediately from representation (4) and commutation relation (3) that

$$Q_{s}|\theta\rangle=0, \tag{6}$$

i.e., the θ vacuum is also supersymmetry-invariant. It is simple to see that the opposite assertion also holds: If the state $|0\rangle$ is not supersymmetry-invariant ($Q_s|0\rangle \neq 0$), then the θ vacuum is also not supersymmetry-invariant ($Q_s|\theta\rangle \neq 0$).

It follows from this discussion and from the relation $P^0 = \frac{1}{4}(Q_s Q_s^+ + Q_s^+ Q_s)$ that if the state $|0\rangle$ is supersymmetry-invariant, then the energy of the θ vacuum does not depend on the parameter θ; i.e., $P^0|\theta = 0$.

The fact that the energy of the θ vacuum for the Lagrangian in (1) does not depend on the parameter θ is a direct consequence of the presence of the Peccei–Quinn symmetry,[8]

$$\psi^a \to \exp(i\alpha j_5)\psi^a$$

for Lagrangian (1).

More complicated supersymmetry modes may be missing in the Peccei-Quinn symmetry (the R symmetry[1]). In general, therefore, the energy of the θ vacuum may depend in a nontrivial way on the parameter θ. The circumstance that the energy of the θ vacuum constructed from the supersymmetry-invariant vacuum of the perturbation theory does not depend on the parameter θ is an extremely nontrivial point.

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