Orientational effect of a surface light wave on liquid crystals

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It is predicted that the nonpropagating light wave appearing with total internal reflection and localized near the interface has a strong effect on a nematic or cholesteric liquid crystal mesophase. The change in the Grandjean structure of the cholesteric liquid crystal is manifested as a change in pitch. On the other hand, the planar structure of the nematic liquid crystal becomes a chiral structure under the action of the wave.

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In recent years, the orientational optical nonlinearity in liquid crystals (LC) has been widely studied.[1–7] Until now, investigations in this region concerned the volume interaction of a light wave with the LC. In this paper, we examine the orientational action of a nonpropagating light wave localized near the boundary on a nematic LC (NLC). Such a wave can be obtained with light incident on a cell containing a NLC through a transparent substrate, whose index of refraction $n_i = \sqrt{\epsilon_i}$ is greater than the index of refraction of the NLC for the ordinary and extraordinary waves. It is localized near the interface within a thickness l, constituting several wavelengths (see below).

Assume the cell is level due to polishing of the surface $z = L$, while the surface $z = 0$ has no effect on the orientation (see Fig. 1). The strain energy per unit surface area of the cell with an inclination by an angle $\theta(z=0) = \beta$ is $\sim\beta^2 K_{22} L^{-1}$ erg/cm^2. The energy of interaction with the light field is of the order of $l\epsilon_a |E|^2 \sin^2(\beta - \beta_1)/16\pi$ erg/cm^2, where β_1 is determined by the orientation of the polarization of the incident wave. Then, for $\beta_1 \neq m\pi/2$, the "giant" nonlinearity with equilibrium value $\beta \sim \epsilon_a |E|^2 \sin 2\beta_1 Ll/K_{22}$ occurs. For $\beta_1 = 0$, reorientation arises only when the threshold with respect to $|E|^2$ is exceeded (Fréedricksz transition). At first glance, both of these effects are weaker than the volume effects in Refs. 1 and 7 by a factor L/l.

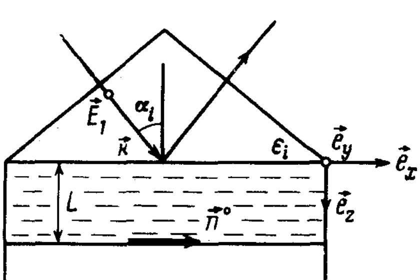

FIG. 1. A cell containing a liquid crystal with a level orientation in which the upper substrate has an index of refraction $n_i = \epsilon_i^{1/2}$ that exceeds the index of refraction of the nematic crystal. The polarization of the incident wave $\mathbf{E}_i$ is parallel and oriented perpendicular to the plane of the figure. The lower substrate orients the director along the direction $\mathbf{n}^0$.

However, a calculation shows that there are other dimensionless numerical factors that greatly increase the effect of surface action. In addition, for a planar cell the Frank constant K_{22} is usually several times smaller than K_{11} and K_{33}.

Let us proceed to the calculation. Let the electric field intensity vector $\mathbf{E}_{i\,\mathrm{mat}}$ of the monochromatic wave incident on the NLC layer be perpendicular to the plane of incidence and related to the complex amplitude $\mathbf{E}_i$ by the relation $\mathbf{E}_{i\,\mathrm{mat}} = \{\mathbf{E}_i \exp(i\omega t - i\mathbf{kr}) + \mathbf{E}_i^* \exp(-i\omega t + i\mathbf{kr})\}$. Using k, we shall define the unit vectors of a Cartesian coordinate system as follows: $\mathbf{e}_y = [\mathbf{k}\mathbf{e}_z]/|[\mathbf{k}\mathbf{e}_z]|$, $\mathbf{e}_x = [\mathbf{e}_y \mathbf{e}_z]$, where $\mathbf{e}_z$ is perpendicular to the plate of the cell (see Fig. 1). The angle between the $\mathbf{n}$ and the x axis, which is actually the line of intersection of the plane of incidence of the light and the interface surface, is denoted by θ, $\mathbf{n} = \{n_x, n_y\} = \{\cos\theta(z), \sin\theta(z)\}$. In the general case of arbitrary mutual orientation of the plane of incidence and the director in the NLC, two refracted waves arise, namely, the ordinary and extraordinary waves, which become nonuniform for certain values of the angle of incidence a_i. The variational equation describing the interaction of a light wave with the NLC has the form[3,8]

$$K_{22}\frac{d^2\theta}{dz^2} + \frac{\epsilon_a}{16\pi}\left\{\sin 2\theta\,(|E_y|^2 - |E_x|^2) + \cos 2\theta\,(E_x E_y^* + E_x^* E_y)\right\} = 0, \tag{1}$$

where K_{22} is Frank's constant and $\epsilon_a = \epsilon_{\parallel} - \epsilon_{\perp}$ is the anisotropy of the dielectric constant of the NLC.

We shall first examine the case in which the unperturbed orientation of the director lies in the plane of incidence of the wave, i.e., $\theta(z = L) = \theta_0 = 0$. Using the expressions for the light field in Ref. 145 and linearizing Eq. (1), we obtain

$$\frac{d^2\theta}{dz^2} + \kappa^2\theta\exp(-z/l) = 0, \tag{2}$$

where

$$\kappa^2 = \frac{\epsilon_a}{2\pi K_{22}}\,\frac{\eta_i^2}{\eta_i^2 + |\eta_{io}|^2}\,\frac{\epsilon_i^2|\eta_{io}|^2 + \epsilon_{\perp}(\epsilon_{\perp}\epsilon_{\parallel})^{1/2}\eta_i^2}{\epsilon_i^2|\eta_{io}|^2 + \epsilon_{\perp}\epsilon_{\parallel}\eta_i^2}\,|\mathbf{E}_i|^2, \tag{3}$$

$$\eta_i^2 = \epsilon_i\cos^2 a_i, \quad \eta_{io}^2 = \epsilon_{\perp} - \epsilon_i\sin^2 a_i, \quad \eta_{ie}^2 = \epsilon_{\perp}^{-1}(\eta_{io}^2 + \epsilon_a\epsilon_i\sin^2 a_i\sin^2\theta),$$

$$l = \lambda/2\pi(|\eta_{io}| + |\eta_{ie}|).$$

We shall seek a solution of Eq. (2) in the form $\theta(z) = A(L - z) + \delta\theta(z)$, where A is a constant, and $|\delta\theta(z)| \ll |Az|$. Specifying, in addition, the boundary condition $d\theta/dz|_{z=0} = 0$ (the condition that the surface of the NLC at $z = 0$ is "free"), we see from (2) that $\kappa^2 = (lL)^{-1}$. From here we determine the power density threshold for the surface light-induced Freedricksz transition (SLFT)

$$P_{\mathrm{thresh}}^{\mathrm{SLFT}} = \frac{c n_i |\mathbf{E}_i|^2_{\mathrm{thresh}}}{8\pi} = \frac{c n_i K_{22}}{4\epsilon_a l L}\,\frac{\eta_i^2 + |\eta_{io}|^2}{\eta_i^2}\,\frac{\epsilon_i^2|\eta_{io}|^2 + \epsilon_{\perp}\epsilon_{\parallel}\eta_i^2}{\epsilon_i^2|\eta_{io}|^2 + \epsilon_{\perp}(\epsilon_{\perp}\epsilon_{\parallel})^{1/2}\eta_i^2} \tag{4}$$

Comparing this quantity with the magnitude of the threshold power densities[8] for the light-induced Freedricksz transition (LFT) with volume interaction of wide beams with the NLC gives

$$\frac{P_{\text{thresh}}^{\text{LFT}}}{P_{\text{thresh}}^{\text{SLFT}}} = 4\pi^2 \frac{K_{33}}{K_{22}} \left(\frac{\epsilon_{\|}}{\epsilon_i} \right)^{1/2} \frac{l}{L} \frac{\eta_i^2}{\eta_i^2 + |\eta_{io}|^2} , \tag{5}$$

where, for simplicity, we assumed that $\epsilon_a/\epsilon_{\|} \ll 1$. Setting $l/L = 10^{-2}$, $\epsilon_i = 1{,}8$, $\epsilon_{\perp} = 1{,}54$, $\lambda = 0.5\,\mu$m, $L = 50\,\mu$m, and $K_{33}/K_{22} \approx 3.3$ (PAA nematic), we obtain $P_{\text{thr}}^{\text{LFT}}/P_{\text{thr}}^{\text{SLFT}} \approx 1.2$. Thus we see that even if the volume interaction of light with the NLC in the presence of SLFT is a factor of 10^{-2} less than for volume LFT, the thresholds of both transitions turn out to be nearly equal. This is attributable to the following circumstances: 1) The distortions of the director field in the presence of SLFT have a twisting character and are most easily realized; 2) in the presence of total internal reflection, the intensity of the light field in the medium is two times larger than the intensity of the incident wave; 3) and, finally, the threshold for SLFT also decreases the "free" nature of one of the surfaces of the NLC cell.

The stationary structure of the SLFT above threshold can be investigated analytically for $\epsilon_a|\epsilon_{\|} \ll 1$. In this case, the variational equation for the director has the form

$$\frac{d^2\theta}{dz^2} + \frac{1}{2}\kappa^2 \sin 2\theta \exp(-z/l) = 0 . \tag{6}$$

Using the fact that $Al \approx \theta(0) l/L \ll 1$, for values of $\theta(0)$ that are not too high, we obtain from (6)

$$\frac{\sin 2AL}{2AL} = \frac{1}{\kappa^2 lL} = \frac{|\mathbf{E}_i|^2}{|\mathbf{E}_i|^2_{\text{thresh}}} = \rho . \tag{7}$$

For $\rho \leqslant 1$, Eq. (7) has a nontrivial solution, which can easily be found graphically. For example, $AL \sim 1$ radians for $\rho = 0.5$.

If the unperturbed orientation of the director forms some angle $\theta_0 \neq 0$ with the plane of incidence of the wave, the surface interaction that was examined by us has a threshold-free character and is analogous to the giant optical nonlinearities which appear with oblique propagation of the extraordinary light wave in NLC. In first order with respect to the field intensity and in the approximation linear with respect to θ, we have from (6)

$$\delta\theta(z=0) = \theta(z=0) - \theta_0 = \frac{1}{2} \frac{|\mathbf{E}_i|^2}{|\mathbf{E}_i|^2_{\text{thresh}}} \sin 2\theta_0 . \tag{8}$$

If, in the figure illustrating the cell, a cholesteric liquid crystal occurs instead of the NLC, the effects that we are examining reduce to a change in the pitch of the helix $\delta q = q - q_0$. The effect is also described by the equations obtained above with the substitution $A \to \delta q$ and taking into account that $\theta_0 = q_0 L$.

The phenomena predicted above make it possible to study LC in the presence of strong volume scattering and in a frequency range where strong absorption occurs.

These effects can easily be studied, for example, by observing the rotation of the plane of polarization of a probing beam propagating in the cell. It would be very interesting to study the orientational interaction of surface plasmon waves, arising at a metal-insulator boundary under certain conditions, with LC.

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