Negative magnetoresistance in semiconductors in the hopping conduction region

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It is shown that negative magnetoresistance in the hopping conduction regime with variable hopping length can be explained by including the displacement of the metal-insulator transition point in an external magnetic field. This explanation can be verified by studying the dependence of the magnitude of the magnetoresistance on the angle between the field and the crystal axes in strongly anisotropic conductors such as deformed Ge and Si.

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In recent years, great progress has been achieved in understanding the negative magnetoresistance (NMR) of disordered conductors in the metallic conductivity region.[1–4] This phenomenon is attributable to the fact that the magnetic field suppresses quantum localization corrections in the Drude equation for the conductivity.[5,6] Numerous results of the theory have been confirmed experimentally. Experiments show, however, that negative magnetoresistance does not disappear with the transition from metallic conductivity to activation conductivity. This cannot be explained on the basis of the existing theory of magnetoresistance in the hopping conduction region, which takes into account only the appearance of an additional potential barrier for tunneling between centers of localization in a magnetic field and leads to a positive magnetoresistance.[7]

In this paper we demonstrate that when a magnetic field H is applied, the localization length, ξ, which decreases the resistance (negative magnetoresistance) can increase. The point is that Anderson localization is intrinsically not a single center phenomenon and the wave function of the electron is formed as a result of its interaction with different centers. The tunneling probability in this case is determined by the localization length ξ, which depends not only on the barrier height but also on the proximity to the metal-insulator transition point. For impurity concentrations n close to the critical concentration n_c, the localization length satisfies

$$\xi \sim (n_c - n)^{-\nu} \gg a_B , \tag{1}$$

where ν is the critical exponent of the conductivity and correlation length. Current experimental data indicate that $\nu \simeq 1/2$,[8] where $a_B = \hbar^2/m^*e^{*2}$ is the Bohr radius of the impurity in the semiconductor. It was indicated in Ref. 9 that the mobility threshold n_c depends on the magnetic field

$$\frac{n_c(H) - n_c(0)}{n_c(0)} = A\left(\frac{eH}{\hbar c}\, n_c^{-2/3}\right)^{1/2\nu} \tag{2}$$

Here A is a number of the order of unity. It is natural to assume that for negative magnetoresistance, n_c in the metallic conductivity region will move into the region with lower concentrations ($A < 0$). If this hypothesis is adopted, then according to (1) $\xi(H) - \xi(0) > 0$.

In the region of variable hopping-length hopping conductivity (VHLHC), the specific resistance is determined by the equation

$$\rho(T) = \rho_0 \exp\left\{\left(\frac{T_0}{T}\right)^{1/a}\right\}; \quad T_0 \sim \xi^{1-a}. \tag{3}$$

Here α can be equal to 2 or 4, depending on whether or not the Coulomb gap is important.[7] If $[n_c(0) - n_c(H)]/n_c(0) \ll 1$, then expanding the exponent in (3) with respect to this parameter and using (1) and (2) we obtain

$$\ln\frac{\rho(T,H)}{\rho(T,0)} = B\left(\frac{eH}{\hbar c} n^{-2/3}\right)^{1/2\nu} \ln\frac{\rho(T)}{\rho_0}, \qquad B = -A\nu\frac{1-a}{a}. \tag{4}$$

As can be seen from (4), the temperature dependence of $\ln[\rho(T,H)/\rho(T,0)]$ is determined by the factor $\ln[\rho(T)/\rho_0]$, which can be experimentally determined independently, while the exponent of the magnetic field is a universal quantity and less than 2 (apparently, $1/2\nu \cong 1$). We note that (4) is valid in the VHLHC region independently of the magnitude of α, on which the coefficient B depends. A similar H and T dependence of the magnetoresistance was observed experimentally.[10]

Measurements of the dependence of the negative magnetoresistance in the VHLHC region on the orientation of H could be an important check of the theory. If the carrier spectrum is strongly anisotropic (for example, in deformed n-Ge), then the shift in the threshold depends on the angles between the direction of H and the axes of the ellipsoid of the diffusion coefficients D_{ij}. Indeed, if in the region of metallic conductivity $D_\parallel$ and $D_\perp$ ($D_{zz} = D_\parallel$) differ considerably, then we can perform the transformation of coordinates given by $z' = z(D_\parallel/D_\perp)^{1/3}$ and $(x', y') = (x, y)(D_\perp/D_\parallel)^{1/6}$, so that in the coordinates (x', y', z') diffusion becomes isotropic with diffusion constant $D_a = (D_\parallel D_\perp^2)^{1/3}$. The magnetic field H in this case transforms into $H' = HD_c/D_a$, where $D_c = [D_\perp/D_\perp \cos^2\theta + D_\parallel \sin^2\theta)]^{1/2}$ and θ is the angle between H and the z axis (see Ref. 3). The isotropic problem contains a displacement threshold and its shift in a magnetic field is determined by expression (2) replacing H by H', i.e., the displacement of the threshold depends on the angle θ. In the VHLHC region, this leads to the following dependence of $\ln[\rho(T,H)/\rho(T,0)]$ on θ:

$$\ln\left[\rho(T,H,\theta)/\rho(T,0)\right]\Big/\ln\left[\rho(T,H,0)/\rho(T,0)\right]$$

$$=\left[1+\frac{D_{\parallel}-D_{\perp}}{D_{\perp}}\sin^2\theta\right]^{1/4\nu} \tag{5}$$

The quantities $D_{\parallel}$ and $D_{\perp}$ (5) correspond to the metallic region in the vicinity of the metal-insulator transition. $D_{\parallel}/D_{\perp}$ apparently varies little in the critical region, so that this ratio can be determined from measurements of the magnetoresistance in the metallic-conductivity region (see Refs. 3 and 11).

Just as for magnetoresistance in the metallic region,^{3} the magnitude of the effect discussed here depends on the time τ_{ϕ} over which the electron makes a transition into a quantum state that is incoherent with respect to the initial state. In the VHLHC region, this time is equal to the expectation time for a hop: $\tau_{\phi}\sim\exp(T_0/T)^{1/\alpha}$. In addition to incoherent hops over a large distance, the electron has time to perform h coherent hops to the closest lying centers. The probability for one such transition is $w\sim\exp[-1/\xi n^{1/3}]$, while the average number of hops can be determined from the condition $\tau_{\phi}w^{\langle k\rangle}\sim 1$, from which $\langle k\rangle\sim(T_0/T)^{1/\alpha}\xi n^{1/3}$. Part of the trajectories formed by coherent hops returns to the initial center, which causes interference of waves that wander along different trajectories. This interference participates, together with the Coulomb field, in the formation of the localization length ξ. A magnetic field strongly affects this interference if the magnetic flux through a typical closed trajectory is comparable to the quantum flux $\Phi_0=2\pi\hbar c/2e$:

$$\frac{eH}{\hbar c}\left(\frac{T_0}{T}\right)^{1/a}\xi n^{-1/3}\gg 1\ , \tag{6}$$

It is important to note that negative magnetoresistance should be expected in the VHLHC region when $w\tau_{\phi}\gg 1$, but not in the region of ϵ_3 conductivity when $w\tau_{\phi}\sim 1$. If condition (6) is not satisfied in the VHLHC region, then negative magnetoresistance is described by the equation

$$\ln\frac{\rho(T,H)}{\rho(T,0)}=\tilde{B}\left(\frac{eH}{\hbar c}n^{-2/3}\right)^2\left[\left(\frac{T_0}{T}\right)^{1/a}\xi n^{1/3}\right]^{(4-1)/2\nu}\ln\frac{\rho(T)}{\rho_0}\ . \tag{7}$$

Positive magnetoresistance, which plays the same role in this problem as the classical magnetoresistance in the metallic region, is the determining factor in the high-field region.^{7,12}

The effect of spin-orbit interaction, the complex structure of the valence band, and intervalley relaxation on the negative magnetoresistance in the VHLHC region will be examined in another paper.

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[1] B. L. Altshuler, D. E. Khmel’nitskiĭ, A. I. Larkin, and P. A. Lee, Phys. Rev. B **22**, 5142 (1980).

[2] A. Kawahata, J. Phys. Soc. Jpn. **49**, 628 (1980).

[3] B. L. Al’tshuler, A. G. Aronov, A. I. Larkin, and D. E. Khmel’nitskiĭ, Zh. Eksp. Teor. Fiz. **81**, 768 (1981) [Sov. Phys. JETP **54**, 411 (1981)].

[4] B. L. Al’tshuler, A. G. Aronov, A. I. Larkin, and D. E. Khmel’nitskiĭ, in: Kvantovaya teoriya tverdykh tel (Quantum Theory of Solids), Ed. by I. M. Lifshitz, Mir, Moscow, 1982.

[5] P. W. Anderson, E. Abrahams, and T. V. Ramakrishnan, Phys. Rev. Lett. **43**, 718 (1979).

[6] L. P. Gor’kov, A. I. Larkin, and D. E. Khmel’nitskiĭ, Pis’ma Zh. Eksp. Teor. Fiz. **30**, 248 (1979) [JETP Lett. **30**, 338 (1979)].

[7] B. I. Shklovskiĭ and A. L. Éfros, Élektronnye svoĭstva legirovannykh poluprovodnikov (Electronic Properties of Doped Semiconductors), Nauka, Moscow, 1979.

[8] T. F. Rosenbaum, K. Andres, G. A. Thomas, and P. A. Lee, Phys. Rev. Let. **46**, 568 (1981).

[9] D. E. Khmel’nitskiĭ and A. I. Larkin, Solid State Commun., **39**, 1069 (1981).

[10] A. N. Ionov, I. S. Shlimak, and A. L. Éfros, Fiz. Tverd. Tela **17**, 2763 (1975) [Sov. Phys. Solid State **17**, 1835 (1975)].

[11] A. N. Ionov and I. S. Shlimak, Pis’ma Zh. Eksp. Teor. Fiz. **35**, 160 (1982) [JETP Lett. **35**, 196 (1982)].

[12] B. I. Shklovskiĭ, Pis’ma Zh. Eksp. Teor. Fiz. **36**, 43 (1982) [JETP Lett. **36**, 51 (1982)].

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