Exponential dependence of the cross sections for deep-inelastic reactions on the quark's transverse momentum

B. L. Ioffe
Institute of Theoretical and Experimental Physics

(Submitted 5 July 1982)
Pis'ma Zh. Eksp. Teor. Fiz. **36**, No. 5, 160–162 (5 September 1982)

It is shown on the basis of the sum rules of quantum chromodynamics that the cross sections for the production of a quark (or of hadron jets) with a transverse momentum $k_\perp$ in deep-inelastic reactions depends exponentially on $k_\perp^2$ if $k_\perp$ is not too large.

PACS numbers: 12.35.Eq, 11.50.Li

The cross sections observed experimentally for the production of hadrons with a transverse momentum $k_\perp$ in deep-inelastic lepton-hadron collisions depends exponentially on $k_\perp^2$, $d\sigma/dk_\perp^2 \sim \exp(-ak_\perp^2)$, if this momentum is not very large, $k_\perp \sim 1$ GeV. This fact has yet to be explained theoretically on the basis of quantum chromodynamics; it is instead incorporated in the theory in an *ad hoc* way. In this letter we show that an exponential $k_\perp^2$ dependence of the cross sections for the production of a quark with a transverse momentum $k_\perp \sim 1$ GeV in deep-inelastic reactions follows from the

 0021-3640/82/170198-04$01.00

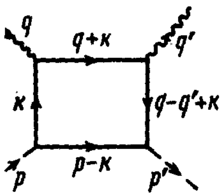

FIG. 1.

sum rules of quantum chromodynamics. Since a quark produced in a deep-inelastic reaction gives rise to a hadron jet, it follows that the dependence is the same for hadron jets.

We begin with the quantum-chromodynamics sum rules proposed by Shifman *et al.*,[1] which have been used successfully to calculate meson masses,[1,2] baryon masses,[3,4] and, through generalizations of these masses, meson widths and form factors.[5,6] We consider the model problem of the scattering of a scalar virtual photon with a momentum q by a scalar pion with a momentum p. We assume $q^2, p^2 < 0$ and $|p^2| \sim 1$ GeV^2 and $|q^2| \gg |p^2|$. Under these conditions the scattering amplitude for the process corresponds to a zeroth-approximation diagram (Fig. 1) in quantum chromodynamics, where the external lines correspond to scalar quark currents. At virtualities $|p^2| \sim 1$ GeV^2, the corrections $\sim \alpha_s$ are small, and the power-law corrections, although numerically important, cannot alter the qualitative exponential behavior. Both can accordingly be ignored.

For the jump in the amplitude $T(r, r', q^2, q'^2, s, t)$ in the s channel,

$$W(r, r', q^2, q'^2, s, t) = [T(r, r', q^2, q'^2, s + i\epsilon, t) - T(r, r', q^2, q'^2, s - i\epsilon, t)] / 2i,$$

where $r = -p^2$ and $r' = -p'^2$, we can write a double dispersion relation in r and r':

$$W(r, r', q^2, q'^2, s, t) = \int_0^\infty \int_0^\infty \frac{\rho(p^2, p'^2, q^2, q'^2, s, t)}{(p^2 + r)(p'^2 + r')} dp^2\, dp'^2, \tag{1}$$

where ρ is the double discontinuity of the function W in p^2 and p'^2, which is found by replacing the propagators by δ functions in the case of the diagram in Fig. 1.

The subtractive terms in (1) are of the form $P_1(r)F_1(r') + P_2(r')F_2(r)$, where P_1 and P_2 are polynomials (the dependence on the other variables is not indicated). We evaluate the left side of (1) in quantum chromodynamics with the help of the diagram in Fig. 1, while we approximate the right side by a sum over the physical states. To avoid the subtractive terms and to suppress the contribution of high-lying states on the right side of (1), we add to (1) a double Borel transformation over r and r':

$$\hat{B}_{M^2} f(r) = \lim (r^{n+1}/n!)(-d/dr)^n f(r),$$

as $r \to \infty$, $n \to \infty$, and $r/n \to M^2$. Setting $M^2 = M'^2$, we find

$$\hat{B}_{M^2} \hat{B}'_{M'^2} W \Big|_{M^2 = M'^2} = \int_0^\infty \int_0^\infty dp^2\, dp'^2 \exp[-(p^2 + p'^2)/M^2] \times \rho(p^2, p'^2, q^2, q'^2, s, t). \tag{2}$$

Since we are interested in the imaginary part of the forward scattering amplitude, we set $q^2 = q'^2$ and $t = 0$. The diagram in Fig. 1 is finite at $t = 0$. Consequently, although quark diagrams are generally singular at $t = 0$, it is legitimate at least qualitatively to extrapolate to the point $t = 0$ in the present case. Evaluation of the diagram in Fig. 1 yields

$$\rho\,(p^2; p'^2; q^2; s) \sim \int dk_\perp^2 \,\delta\,[p\,x\,(1-x) - k_\perp^2]\,\delta\,(p^2 - p'^2), \tag{3}$$

so that, by virtue of (2),

$$\hat{B}_{M^2}\,\hat{B}'_{M^2}\,W \sim \int dk_\perp^2 \,\exp[-2k_\perp^2/M^2\,x\,(1-x)] \tag{4}$$

(we are omitting the coefficient of the exponential function). Here $x = Q^2/2\nu$ is the scaling variable, where $Q^2 = -q^2$ and $s \approx 2\nu - Q^2$. On the other hand, by saturating the right side of (2) with low-lying hadron states in the p and p' channels (with a scalar pion), we can put (2) in the form

$$\hat{B}_{M^2}\,\hat{B}'_{M^2}\,W \sim e^{-2\,m_\pi^2/M^2} \int dk_\perp^2\, d\sigma\,(Q^2; x,\, k_\perp^2)/d\,k_\perp^2\ , \tag{5}$$

where $d\sigma/dk_\perp^2$ is the cross section for the scattering of a virtual photon by a pion, accompanied by the production of a quark with a transverse momentum $k_\perp$. Equating (4) and (5), and assuming a local duality, i.e., assuming that the equality of the integrals implies the equality of the integrands, we find

$$\frac{d\,\sigma\,(Q^2; x,\, k_\perp^2)}{d\,k_\perp^2} \sim \exp\left[-\frac{2\,k_\perp^2}{M^2\,x\,(1-x)}\right] \tag{6}$$

The range of applicability of this approach can be seen from (6): The argument of the exponential function must not be very large, i.e., $k_\perp^2$ must not be too large, and x must not be approximately equal to 0 or 1; otherwise, the corrections which we have ignored will become important.

The assumption of a local duality is a generalization of the hypothesis of quark-hadron duality, according to which the integral of the physical states over some region of the mass spectrum is the same as the integral of the quark states over the same region. This hypothesis works well both for the mass operator[1-4] and for vertex functions,[5] so its generalization seems highly plausible.

An analysis of deep-inelastic eN scattering, observed experimentally, could be completely analogous to the analysis above in the model problem. The scattering of a virtual photon by a nucleon corresponds to the quark diagram in Fig. 2, where p and p'

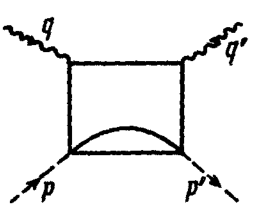

FIG. 2.

are the momenta of the three-quark currents with the quantum numbers of the nuclei.[3] For the double spectral function we find an expression which differs from (3) in that $\delta\,[p^2x(1-x)-k_\perp^2]$ is replaced by $\theta\,[p^2x\times(1-x)-k_\perp^2]$ (in addition to the factors which constitute the coefficient of the exponential function). As a result we find for $d\sigma/dk_\perp^2$ the same exponential behavior as is described by (6).

For scattering by a meson the quantity M^2 in (6) is $M^2 \approx 1.2$ GeV^2; for scattering by a nucleon it is $M^2 \approx 2$ GeV^2 (the values of M^2 for the double Borel transformation are roughly twice as large as those in the polarization operator[5]).

Expression (6) predicts a nontrivial x dependence of the transverse-momentum distributions. An experimental test of this expression would be of major interest.

[1]M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 385, 448 (1979).
[2]L. J. Reinders, H. R. Rubinstein, and S. Yazaki, Nucl. Phys. **B186**, 109 (1981).
[3]B. L. Ioffe, Nucl. Phys. **B188**, 317 (1981); **B191**, 541 (Errata).
[4]V. M. Belyaev and B. L. Ioffe, Preprint ITEP-59, 1982.
[5]B. L. Ioffe and A. V. Smilga, Preprint ITEP-21, 1982.
[6]A. V. Nesterenko and A. V. Radyushkin, Pis'ma Zh. Eksp. Teor. Fiz. **35**, 395 (1982) [JETP Lett. **35**, 488 (1982)].

Translated by Dave Parsons
Edited by S. J. Amoretty