Dynamic model of the real part of the scattering amplitude at zero angle

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The real part of the elastic forward scattering amplitude is examined on the basis of the diffraction dissociation model. In this picture, the positive sign of the real part observed at high energies indicates that $\mathrm{Re}\, f(0)$ is an entirely peripheral effect, a halo. This conclusion agrees with the available experimental data.

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The real part of the hadron-hadron elastic forward scattering amplitude at high energies is an important characteristic of the S matrix, which has never been given a clear dynamic explanation. While the dynamics of the appearance of the imaginary part is followed well both quantitatively and qualitatively in models based on the multiperipheral representations, which in turn have received a space-time interpretation in the quark-parton picture,[1] and then in the framework of QCD,[2–4] the real part is determined from the imaginary part based on dispersion relations. Because of their

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integral nature, dispersion relations do not yet make it possible to understand the dynamics of the real part based on the familiar multiperipheral picture of the imaginary part.

The main property of $\text{Re}\,f(0)$ consists of the fact that in the region $E_{\text{lab}} \gtrsim 10^2$ GeV the quantity $\rho = \text{Re}\,f(0)/\text{Im}\,f(0)$ changes sign, passing from negative values with $E_{\text{lab}} < 10^2$ GeV to positive values $\rho \sim +0,1$. Experimental values are currently available for ρ in the region 10^2–10^3 GeV only for pp and $\pi\bar{p}$ scattering,[5,6] but the positiveness of $\text{Re}\,f(0)$ at high energies apparently has a universal nature, since it follows from dispersion relations in connection with the well-known increase in σ_{tot}.

The scattering amplitude in the impact parameter (b) representation has the form $f(b) = (\bar{\chi}(b)\exp 2i\bar{\delta}(b) - 1)/2i$, where $0 < \bar{\chi}(b) \leq 1$ is the transparency (fraction of surviving particles). Since $\text{Re}\,f(0) = \bar{\chi}(b)\sin 2\bar{\delta}(b)/2$, while $\text{Im}\,f(b) = (1 - \cos 2\bar{\delta}(b))\bar{\chi}(b)/2$ the problem of the real part of $f(b)$ reduces to the problem of the mechanism for the appearance of phase shift $\bar{\delta}(b)$ in high-energy scattering.

Our analysis is based on an idea dating back to the papers by Feĭnberg and Pomeranchyk[7] as well as Cood and Walker[8] concerning the fact that the components of the wave function of the incident hadron corresponding to different internal coordinates of the constituents are absorbed differently by the target. According to the description in Ref. 10, the target acts like a unique filter which destroys the equilibrium wave function of the incident particle. Equilibrium is restored some time after passage through the target (hadronization time $t_a \cong E/m^2$).[1,9] At the same time, the components of the initial wave function of the hadron, which correspond to nonequilibrium small transverse distances between the constituents (large $k_\perp$), decay, giving rise to diffraction dissociation.[10]

It is very significant that each filtered component of the wave function of the incident particle has some probability amplitude for decaying into a state corresponding to the wave function of the initial particle. In this case, however, there is a phase shift which depends on the coordinates of the constituents of the incident particle that passes through the target.

We shall represent the wave function of the incident particle as a Fock column matrix, defined in simultaneous dynamics on the light cone.[10,11] The internal coordinates $\xi(x_i, k_{\perp i})$ of such a wave function are the transverse momenta $k_{\perp i}$ (or coordinates $r_{\perp i} = 1/k_{\perp i}$) of the constituents and the fraction of the longitudinal momentum on the light cone carried off by the constituents $x_i = k_{0i} + k_{3i}/p_0 + p_3$, where p_3 is the momentum of the incident particle in the system $p_1 = p_2 = 0$. At high energies, the transparency χ and phase shift δ can be written as functions of energy, impact parameter and internal coordinates of the incoming particle: $\chi = \chi(E,b,\xi)$ and $\delta = \delta(E,b,\xi)$. On the other hand, the wave function of the incident particle $\psi(\xi)$ is normalized: $\int |\psi(\xi)|^2 d\xi = 1$. For scattering at zero angle, the helicity is conserved and for this reason we shall ignore quark spins. In this case $\chi(E,b,\xi)$ and $\delta(E,b,\xi)$ are simple numbers and we can determine the expected value of the physical quantities $\bar{\chi}(E,b)$ and $\bar{\delta}(E,b)$: $\bar{\delta}(E,b) = \int d\xi\, |\psi(\xi)|^2 \delta(E,b,\xi)$ and a similar expression for $\chi(E,b)$. Let us form from the incoming particles a very narrow wave packet ($\Delta E \cong m$) with average energy $E = (m^2 + p^2)^{1/2}$ ($p = p_3$) and average phase ($pz - Et$) and let us follow the interaction of one of the Fock configurations with target thickness $\sim 1/m$. Let the packet $[\psi(\xi)$

exp $i(pz - Et)]$ reach the target at $t = 0$, $z = 0$ and interact with it. The initial amplitude of the Fock configuration $\psi(\xi)$ decreases as a result of the interaction and becomes $\eta_1(E,b,\xi)\psi(\xi)$. For this reason, after a time $t_a \cong E/m^2$ this nonequilibrium configuration decays, after which the wave function will contain a fraction of the initial wave function with amplitude $\eta_2(E,b,\xi)\eta_1(E,b,\xi)\psi(\xi)$ (η_2 depends on E and b, since for $b \rangle$ the interaction radius $\eta_1 = \eta_2 = 1$). Thus each configuration ξ contains a fraction $\psi^*(\xi)\eta_1\eta_2\psi(\xi) = |\psi(\xi)|^2\eta(E,b,\xi) = \chi(E,b,\xi)$ of the surviving particles in the beam and

$$\bar{\chi}(E,b) = \int d\xi\, |\psi(\xi)|^2 \eta(E,b,\xi) \leqslant 1\,. \tag{1}$$

The quantity $|\psi(\xi)|^2\delta(E,b,\xi)$ is equal to $\chi(E,b,\xi)\delta(\xi)$, where $\delta(\xi)$ is the phase shift caused by the passage of the configuration ξ. We shall describe the configuration ξ by the quantities x and $k_\perp$, the fraction of the longitudinal momentum and the transverse momentum carried away by one of the quarks (q); the corresponding quantities for the remaining quarks and gluons (q,g) will be $1-x$ and $-k_\perp$. Then, the phase of the configuration ξ at the time of its decay is

$$(xpz_a + (1-x)pz_a - (E_q + E_{qg})t_a = [p - (k_\perp^2 + x^2p^2)^{1/2} - (k_\perp^2 + (1-x)^2p^2)^{1/2}]\frac{E}{m^2}$$

(we ignore the mass of light quarks and the mass of the system q,g). If $b >$ the radius of interaction, then the packet passing through the target at time t_a would have the average phase $(p - (m^2 + p^2)^{1/2})(E/m^2)$. Thus, the phase shift, which is caused by the passage of the configuration ξ, is

$$\delta(\xi) = -\left[(k_\perp^2 + x^2p^2)^{1/2} + (k_\perp^2 + (1-x)p^2)^{1/2} - (m^2 + p^2)^{1/2}\right]\frac{E}{m^2}$$

$$= \frac{1}{2}\left(1 - \frac{k_\perp^2}{m^2(1-x)x}\right)$$

Finally,

$$\bar{\delta}(E,b) = \frac{1}{2}\int d\xi\left(1 - \frac{k_\perp^2}{m^2(1-x)x}\right)|\psi(\xi)|^2\eta(E,b,\xi) = \frac{1}{2}\left(1 - \frac{\bar{k}_\perp^2}{m^2(1-\bar{x})\bar{x}}\right)_{E,b}$$

$$\times \int d\xi\, |\psi(\xi)|^2\eta(E,b,\xi) = \frac{1}{2}\left(1 - \frac{\bar{k}_\perp^2}{m^2(1-\bar{x})\bar{x}}\right)_{E,b}\bar{\chi}(E,b). \tag{2}$$

It is evident that $\bar{\delta}(E,b)$ is proportional to $\bar{\chi}(E,b)$. The quantity $(\bar{k}_\perp^2/m^2(1-\bar{x})\bar{x})_{b,E}$ describes the configuration for which the target is most transparent for the incident particle with given b.

Interchanging the beam and the target and retaining s, we obtain expressions analogous to (1) and (2) for $\bar{\delta}_t(s,b)$ and $\bar{\chi}_t(s,b)$ for the target particles. Analyzing the interaction of two wave packets in the center of mass system, it is easy to verify that the total phase shift is given by $2\bar{\delta}(s,b) = \bar{\delta}_B(s,b) + \bar{\delta}_t(s,b)$. On the other hand, due to the law conservation of energy and momentum, each elastically scattered particle in the beam corresponds to only one elastically scattered particle in the target. For this reason, $\chi_B(s,b) = \overline{\chi}_t(s,b) = \overline{\chi}(s,b)$ and

$$2\bar{\delta}(s,b) = \bar{\chi}(s,b)\left(1 - \frac{1}{2}\frac{\bar{k}_\perp^2}{m_B^2(1-\bar{x})\bar{x}} - \frac{1}{2}\frac{\bar{k}_\perp^2}{m_t^2(1-\bar{x})\bar{x}}\right)_{s,b} \tag{3}$$

The quantity $(\bar{k}_\perp^2/m^2(1-\bar{x})\bar{x})_s$ averaged over the impact parameter for configurations with highest transparency can be estimated from experimental data on ρ. For *pp* scattering

$$\rho = \frac{4\pi}{\sigma_t}\int \bar{\chi}\sin 2\bar{\delta}\, b\, db = \frac{4\pi}{\sigma_t}\int_0^R \bar{\chi}^2\left(1 - \frac{k_\perp^2}{m^2(1-\bar{x})\bar{x}}\right)_{s,b} b\,db$$

$$\cong \frac{2\pi r_{ms}^2}{\sigma_t}\bar{\chi}^2(r_{ms})\left(1 - \frac{\bar{k}_\perp^2}{m^2(1-\bar{x})\bar{x}}\right)_s, \tag{4}$$

where $\chi^2(r_{ms}) = 1 - 4G(r_{ms})$; and $G(r_{ms})$ is the inelastic overlap function at the mean square radius.

Thus, $p > 0$ when $\bar{k}_\perp^2/m^2(1-\bar{x})\bar{x} < 1$. This means that at high energies the survival of an incident particle is related to the transmission of configurations having large transverse distances between constituents: $r_\perp > 2/m$. This conclusion will only be

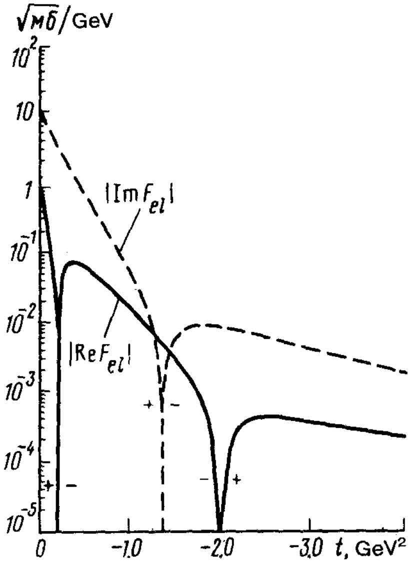

FIG. 1. The real and imaginary parts of *pp* scattering at $\sqrt{s} = 53$ GeV, from Ref. 12.

strengthened if the masses of the quarks and the mass of the system, q,g are taken int account. There arises the question as to whether or not this contradicts the result of QCD, namely that white configurations of the beam with large $k_\perp$ interact weakly with the target, while configurations with small $k_\perp$ are almost completely absorbed.[2–4] The contradiction is eliminated if it is assumed that the transmission of small $k_\perp$ becomes possible at high energies at the periphery. Large $k_\perp$ pass through the target very well: $\eta_1(k_\perp > m) \sim 1$, but decay mainly not along the input channel, but into fragments of diffraction dissociation: $\eta_2(k_\perp > m) \sim 0$; i.e., the target is nontransparent for large $k_\perp$. If passage of small $k_\perp$ becomes possible at the periphery, then subsequently these valence configurations are "dressed" and go over into a state of the initial beam with high efficiency: $\eta(k_\perp \lesssim m/2, b \gtrsim 2/m) \sim 1$.

In this picture, $\mathrm{Re} f(0)$ appears as a highly peripheral effect, a halo. This result agrees with available data on the form of the dependence $\mathrm{Re} f(t)$ and $\mathrm{Im} f(t)$ in pp scattering with $\sqrt{s} = 53$ GeV, obtained from derivative dispersion relations (Fig. 1).[12] Comparing the position of the diffraction minima in $\mathrm{Re} f(t)$ and $\mathrm{Im} f(t)$, we find that the ratio of their radii $\gtrsim 2.5$.

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