Dynamic chaos, Anderson localization, and confinement

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A possible randomization of quantum chromodynamics can give rise to an effect similar to the localization which occurs in disordered macroscopic media. As a result, a discrete energy spectrum corresponding to a linearly increasing potential arises in a quark-antiquark system.

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Discussions of the physical mechanisms for confinement have dealt with various types of ordering of the quantum chromodynamics of the vacuum and also the opposite possibility, corresponding to stochastic properties of the vacuum. For example, Olesen[1] has recently pointed out that an area theorem follows directly from the random nature of the distribution of the color-field flux over the area of the Wilson contour. In this letter we discuss some different, less formal aspects of a possible relationship between a stochastic nature and confinement.

1. Much progress has been achieved toward reaching an understanding of the nature of a random behavior (see Ref. 2, for example). It turns out that the determinate behavior of simple nonlinear systems in classical mechanics may be so complicated, tangled, and essentially unpredictable that it is indistinguishable from a random behavior ("dynamic chaos"). It has recently become clear[3] that the same is true of the nonlinear Yang-Mills classical equations, which, even in a simplified version, reveal an extreme irregularity of the components of the color field as functions of the time.

So far there has been no corresponding study of the quantum-mechanical Yang-Mills equations (or, especially, of the general equations of quantum chromodynamics), although we should expect to find weaker stochastic properties here than in classical mechanics.[4] Nevertheless, the possibility of a distinctive irregularity of the physical quantities of quantum chromodynamics would be difficult to rule out completely. The consequences of such an irregularity are discussed below.

2. Adopting for definiteness a system consisting of a heavy quark and a heavy antiquark, we make the single assumption that, in accordnace with predictions, the potential of the interaction between the particles, V, is a complicated, irregular function which relates the vectors of these particles, $\mathbf{r}$. Ignoring the spin, we may assume that the argument of this function is the quantity $r = |\mathbf{r}|$, since the vacuum is isotropic, and the problem contains no vectors other than $\mathbf{r}$. The radial part of the wave function of the relative motion of particles thus obeys a one-dimensional Schrödinger equation (the one-dimensional nature is important for the discussion below) which takes the following form for the s states in a system of units with $\hbar = m = 1$:

$$\left(d^2/dr^2 + E - V(r)\right)\chi = 0, \qquad \chi = r\psi. \tag{1}$$

If we ignore the distinctions between the radial motion and purely one-dimensional motion, which are of no consequence for the arguments below, we see that this equation corresponds to the typical problem of the one-dimensional motion of a particle in an irregular external field, which has been the object of many studies in the physics of disorder systems.[5,6] A remarkable property of this motion, which is seen in the *one-dimensional case*, is that, regardless of its energy, the particle is localized near a certain point r_0 (Anderson localization). This result means that the envelope of the (irregular) wave function is damped exponentially with distance from r_0 over the localization length $l\ (E)$.

3. The irregular potential is customarily replaced by the random function $V(r)$ with the properties

$$\langle V(r)\rangle = 0, \qquad \langle V(r)\,V(r')\rangle = D(r-r') \tag{2}$$

(we are omitting the regular part of the potential for simplicity). The corresponding localization length generally depends on (2) and on high-order correlators. At energies

$$L^{-2} \ll E \ll R^{-2}, \tag{3}$$

however, the function $l(E)$ becomes a universal function, determined exclusively by $L = [\int_0^\infty drD(r)]^{-1/3}$. R is the correlation radius, over which the function $D(r)$ decays substantially (see Sec. 10 in Ref. 6)[1]:

$$l(E) = L^3E\ . \tag{4}$$

Expression (4) is actually the same as the mean free path calculated in the Born approximation.

The localization effect gives the spectrum of Eq. (1) some unusual properties. On the whole, the spectrum is quasicontinuous (like the set of rational numbers), but the wave functions corresponding to approximately the same energy are localized far from each other. Consequently, a discrete spectrum (with levels separated by a distance determined by the localization length) corresponds to the wave functions localized near a given point r_0.

4. To describe this "local" discrete spectrum and other characteristics of a particle localized near r_0, it is convenient to replace the irregular (random) potential $V(r)$ by the specially selected regular potential $U(r-r_0)$, which gives the correct localization length. Since the motion is semiclassical [the wavelengths of the particle are small in comparison with the localization length according to (3) and (4)], we can determine U simply from the condition that the width of the corresponding classically allowed region of motion agrees with the given value of $l(E)$.

This width is determined by the condition $E = U(r-r_0)$, and the potential U itself is correspondingly determined by the equation $l(U) = |r-r_0|$. For the universal law in (4) we thus find a function which increases linearly in the region $L \ll |r-r_0| \ll L^3R^{-2}$:

$$U(r-r_0) = |r-r_0|/L^3\ . \tag{5}$$

5. Returning to the quark-antiquark system, we see that if these particles are produced close to each other (e.g., in e^+-e^- annihilation), they cannot move apart by distances greater than the localization length.[2)] This assertion is a consequence of

Anderson localization along the coordinate of the relative motion of the particles. An important point is that, since it is localized (near $r_0 = 0$), the quark–antiquark system has a local discrete spectrum ("local" in the sense given above) which corresponds to an effective potential U, which increases linearly with the distance.

These results, as tentative as they are, are a consequence of such simple assumptions that a further study of the relationship between a stochastic nature and confinement seems worthwhile. Unfortunately, such a study would be hindered by the lack of direct data on the stochastic properties of quantum chromodynamics or even the quantum-mechanical Yang-Mills equations.

A more detailed exposition of the question raised here will be published in the Proceedings of the 1982 School of the Institute of Theoretical and Experimental Physics.

[1)]The quantity L represents the amplitude of the potential "surges," while R represents the extend to which the potential is irregular. We are assuming $R \ll L$ (a pronounced irregularity).

[2)]External agents may cause the particles to switch to a state with a different localization point, corresponding to a greater distance between the particles. The probability for such a transition, however, is exponentially small, in accordance with the small overlap of the corresponding wave functions.

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