Oscillation of the transparency of metals in a weak magnetic field

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It is shown that under the conditions of the anomalous skin effect in a weak magnetic field **H**, parallel to the surface of a conductor, effective electrons form not only the skin layer but also a weakly damped component of the electromagnetic field. As a result, the transparency of thin metallic plates in the microwave range oscillates as a function of the magnetic field, while the period of these oscillations with respect to $H^{-1/2}$ is determined by the location characteristics of the Fermi surface.

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Under the conditions of the anomalous skin effect, electrons moving in phase with the wave and interacting with it efficiently form a narrow skin layer with thickness δ. Noneffective electrons, which traverse the skin layer almost normal to the surface of the metal, carry information about it into the bulk of the specimen and are responsible for the weakly damped component of the field, decreasing over distances of the order of the mean free path l.[1] In a strong magnetic field parallel to the metal surface (radius of curvature of the electron trajectory r is much smaller than l), the spiking mechanism for drawing the high-frequency (HF) field into the metal is characteristic.[2] The presence of field spikes leads to an anomalous transparency of thin plates, when the thickness d is a multiple of the extremal diameter of an electron orbit, and to oscillations in the impedance as a function of H.

However, even in weak magnetic fields, when not even one spike formed by electrons with an extremal orbit diameter fits into the thickness of the conductor, the electromagnetic field penetrates into the metal over a considerable distance x and the transparency of thin plates is an oscillating function of H.

In the microwave range in pure specimens ($\omega\tau \gg 1$, ω is the frequency of the electromagnetic wave and τ is the free flight time of an electron) we can realize the case in which the electron does not experience collisions over the flight time of an effective electron through the narrow skin layer, but the phase of the HF field changes repeatedly, i.e., the following inequalities are satisfied:

$$\Delta << \delta << l^2/r; \qquad \Delta \equiv {}^2/_{r_0}\,(v_0/\omega)^2, \tag{1}$$

where Δ is the path traversed by an electron along the direction of propagation of the wave over the period $2\pi/\omega$, r_0 is the radius of curvature of the electron orbit at the point at which it turns into the skin layer, and v_0 is the velocity of an electron at the same point. A characteristic of the region of frequencies ω and magnetic fields examined here is that temporal dispersion is also clearly manifested in the HF field distribution against the background of the distinct spatial dispersion characteristic of the anomalous skin effect. Moving into the bulk of the conductor along a trajectory curved by the magnetic field, an effective electron interacts with the rapidly varying HF field, which for an electron located at a distance $x \gg \Delta$, reverses its orientation $2\sqrt{x/\Delta}$ times. The sharp nonuniformity of the HF field leads to the fact that the energy acquired by the charge carrier turns out to be proportional to a periodic function of the argument $2\sqrt{x/\Delta}$. It is as if the effective electron, in moving from the skin layer into the bulk of the conductor, realizes a nonlinear unfolding of the temporal oscillations of the electromagnetic field into spatial oscillations and, in magnetic fields satisfying conditions (1), there arises a weakly damped HF field component which has a unique coordinate dependence.

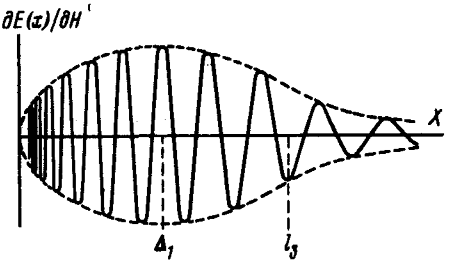

FIG. 1. A schematic diagram of the dependence of the derivative on the amplitude of the weakly damped component with respect to the magnetic field. The bending of the electron trajectories by the magnetic field leads to the appearance of a maximum in the envelope and its slow decrease as $x^{-3/4}$, which for $x > l_3$ is replaced by an exponential decrease of the correction to the Roiter–Sondheimer component.

Determining with the help of the kinetic equation the relation between the HF electric field amplitudes $E(x)$ and the current $j(x)$, it is not difficult to solve Maxwell's equations using perturbation theory and the well-known solution of these equations for $H=0$ as the zero-order approximation.[1] Differentiating with respect to the magnetic field and thereby eliminating from the background part the HF field distribution, we obtain the following expression for $\partial E(x)/\partial H$:

$$\frac{\partial E(x)}{\partial H}=\frac{aE(0)}{H}f(\Delta_e/\delta)\left\{\frac{(x/\Delta_1)^{3/4}}{1-i(x/\Delta_1)^{3/2}}\exp(-2\sqrt{x/l_3})\cos(2\sqrt{x/\Delta_e}+s\pi/4)\right\}. \quad (2)$$

Here α is a quantity of the order of unity, determined by the anisotropy of the Fermi surface; Δ_e is the extremal value of Δ as a function of the projection of the momentum along the direction of the magnetic field p_z; $s=\text{sign}(\partial^2\Delta/\partial p_z^2|_{p_z=p_z^{\text{extr}}})$; $l_3=\Delta_e(\omega\tau)^2\simeq 2l^2/r$; is the damping distance; and, $\Delta_1=\delta^2/\Delta_e$.

The effect being examined is insensitive to the reflection of charge carriers from the boundaries of the specimen and we shall present an expression for the function $f(\Delta/\delta)$ for the case of purely specular reflection of electrons:

$$f(\Delta/\delta)=\Gamma(11/2)(\Delta/\delta)^{11/2}+{}^{\pi}/_{3}\exp\{{}^{i\pi}/_{3}-{}^{\delta}/_{2\Delta}(1+i\sqrt{3})\}, \quad (3)$$

$\Gamma(z)$ is Euler's gamma function. The solution of the problem for arbitrary reflection of electrons from the boundaries of the specimen shows that only the function $f(\Delta/\delta)$ depends on the form of the scattering indicatrix, while the expression in braces in Eq. (2), which describes the damping of the HF field into the bulk of the specimen, remains unchanged.

The presence of a weakly damped component of the HF field (2) leads to oscillations in the transparency of thin metallic plates, whose thickness d is much less than r. The derivative of the coefficient of transmission T of the electromagnetic wave through the plate measured in the experiment is defined by Eq. (2), in which we must set $x=d$ (Fig. 2). It is easy to see that $\partial T/\partial H$ is periodic as a function of $H^{-1/2}$ with period

$$\Delta(1/\sqrt{H})=\left(\frac{2\pi^2 e v_0^2}{c p_0 \omega^2 d}\right)^{1/2}, \quad (4)$$

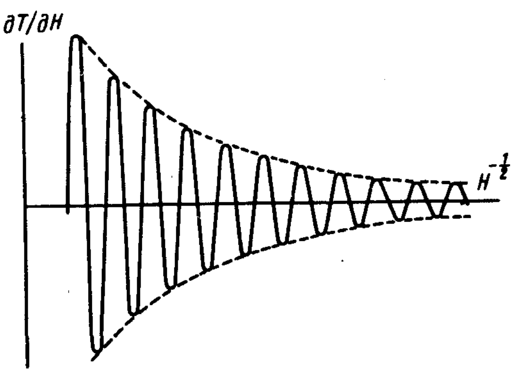

FIG. 2.

where e is the electron charge and c is the velocity of light. It is convenient to observe the effect indicated by measuring the intensity of the electromagnetic wave which passes through the specimen in magnetic fields

$$l \ll r \lesssim l^2/d. \tag{5}$$

Under these conditions, the weakly damped component (2) is the only reason for the oscillatory H dependence, while its amplitude varies little with transmission through a thin metallic layer with thickness d.

In magnetic fields $d < 2r \ll l$, electrons with orbital diameter close to the thickness of the plate form a HF field spike at its boundary opposite to the skin layer. Oscillations in the transparency have a resonant character, while the resonance occurs when the frequency ω is a multiple of the rotational frequency Ω_1 of the electrons indicated along the orbit in a magnetic field. In this case, we have for $E(d)$

$$E(d) = AE(0)\delta/df_1(\Delta/\delta)\,\{1 - \exp(2\pi i\omega/\Omega_1 - 2\pi/\Omega_1\tau)\}^{-1}; \qquad |A| \simeq 1; \tag{6}$$

$$f_1(\Delta/\delta) = \Gamma^2(11/2)(\Delta/\delta)^{11} + 2\pi/3\,\Gamma(11/2)(\Delta/\delta)^{11/2}\exp({}^{i\pi}/_3 - {}^{\delta}/_{2\Delta}(1 + i\sqrt{3}))$$
$$+ \frac{\pi^2}{9}\exp({}^{2\pi i}/_3 - {}^{\delta}/_{\Delta}(1 + i\sqrt{3})). \tag{7}$$

The amplitude of the oscillations described above is small with respect to the parameter Δ/δ and decreases rapidly with decreasing magnetic field. However, the paper of Azbel', which concerns the observation of the cyclotron resonance in massive conductors with $\Omega \ll \omega, r \ll l$, convincingly demonstrates the reality of the new oscillatory effects observed, which permits obtaining important information on the local characteristics of the Fermi surface and the relaxation properties of conduction electrons.

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