Relation between scattering lengths and effective radii in the charged and neutral channels

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The Coulomb correction to the strong scattering length $a_l^{(s)}$ and the effective radius $r_l^{(s)}$ are calculated for states with arbitrary angular momentum l. This allows relating the position and width or near threshold resonances in charged and neutral channels for the lightest nuclei (^{5}He and ^{5}Li, etc.). The Coulomb correction to $a_l^{(s)}$ is especially large for the s wave, while the correction to $r_l^{(s)}$ is large for the p wave, when the contain the large logarithm $\ln(a_B/r_0)$, where a_B is the Bohr radius and r_0 is the range of nuclear forces.

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1. Refs. 1–3, the following equation was used to analyze the spectra of hadronic $p\bar{p}$ and $\Sigma^- p$ atoms

$$\left\{ \lambda + 2\zeta \left[\psi(1-\zeta/\lambda) + \ln\frac{\lambda}{|\zeta|} \right] \right\} \prod_{j=1}^{l} \left(\frac{\zeta^2}{j^2} - \lambda^2 \right) = \frac{1}{a_l^{(cs)}} + \frac{1}{2} r_l^{(cs)} \lambda^2 , \quad (1)$$

which relates the shifts in the atomic levels to the low-energy scattering parameters. Here $\hbar = m = e = 1$, m is the reduced mass, $\xi = -Z_1Z_2$, $\lambda = (-2E)^{1/2}$, and E is the energy in units of $E_C = me^4/\hbar^2$, $a_l^{(cs)}$ and $r_l^{(cs)}$ are the nuclear-Coulombic scattering lengths and effective radius.

It is often worthwhile to extract from the experiment not $a_l^{(cs)}$ and $r_l^{(cs)}$ but the parameters $a_l^{(cs)}$ and $r_l^{(cs)}$, which relate directly to the strong potential V_s with the Coulomb interaction switched-off. The difference between $a_l^{(s)}$ and $r_l^{(cs)}$ is especially large when the potential V_s contains a level close to 0. This situation is realized in np and pp systems, as well as possibly in the $p\bar{p}$ atom.[1] The relation between the parameters $a_l^{(cs)}$ and $a_l^{(s)}$ was first found by Schwinger for $l=0$,[4] and for $l \geqslant 1$ in Ref. 5:

$$\frac{1}{a_l^{(cs)}} - \frac{1}{a_l^{(s)}} = 2\zeta \left[\frac{(2l)!}{2^l \, l!} \right]^2 J_l , \quad (2)$$

where $J_l = \int_0^\infty \chi_l^2(r) dr/r$ for $l \neq 0$,

$$J_0 = \int_0^R \chi_0^2(r) \frac{dr}{r} + \int_R^\infty \frac{\chi_0^2(r)^{-1}}{r} dr - 2C - \ln(2|\zeta|R),$$

$C = 0.5772...$, and χ_l is the wave function when the level appears.[1)]

We note that the relation between *nd* and *pd* scattering lengths were examined in a recent paper within the scope of the three-body problem.[6]

2. We also obtained an expression for the Coulomb correction to the effective radii $r_l^{(s)}$ with arbitrary l. If $l \neq 1$, then

$$r_l^{(cs)}/r_l^{(s)} = 1 + h_l \zeta \, |r_l^{(s)}|^{-\frac{1}{2l-1}} + \dots, \tag{3}$$

where h_l is a dimensionless factor that depends on the potential V_s. A numerical calculation for s states gives: $h_0 = 1.19$, 0.14, and -0.18, respectively, for a rectangular well and for the Hulten and Yukawa potentials. In most case $h_0 > 0$; as a result, the effective radius r_{cs} decreases compared to r_s (in the case $\zeta < 0$, i.e. for Coulombic repulsion). This can be illustrated for *pp* scattering. According to Ref. 7, $r_{pp} = 2.80 \pm 0.02$ fm and $r_{nn} = 2.86 \pm 0.03$ fm. From here $h_0 \approx 0.4$ which is close to the value $h_0 = 0.51$ for an exponential potential. The fact that h_0 is positive also explains the fact that $r_{cs} < r_s$ in the case of $\alpha\alpha$ scattering, which was quoted by Kok.[8]

The case $l = 1$ is a special case. The Coulomb correction to the effective radius in this case is especially large, so that in contrast to (3) it contains a large logarithm:

$$r_1^{(cs)}/r_1^{(s)} = 1 + \frac{4\zeta}{r_1^{(s)}}\left(\ln\left|\frac{\zeta}{r_1^{(s)}}\right| + \beta_1\right), \tag{4}$$

where β_1 is a constant that can be computed and is insensitive to the model of V_s (for example, $\beta_1 = 0.70$ for a rectangular well and $\beta_1 = 0.74$ for a separable Yamaguchi potential, etc.).

In the general case (arbitrary l) $\ln(a_B|r_0)$ arises only in the Coulomb correction to the coefficient ρ_l for k^{2l} in the effective-range expansion:

$$k^{2l+1}\operatorname{ctg}\delta_l = -\frac{1}{a_l} + \frac{1}{2}r_l k^2 + \dots + \rho_l k^{2l} + \dots .$$

In the remaining terms of this expansion, the Coulomb renormalization $\sim r_0/a_B \ll 1$, as in Eq. (3). In this case, with logarithmic accuracy, the Coulomb correction to the coefficient $\rho_l^{(s)}$ has a universal form

$$\rho_l^{(cs)} - \rho_l^{(s)} = 2\zeta\{\ln(|\zeta| r_0) + \text{const}\}. \tag{5}$$

Here r_0 is the range of nuclear forces, and only the constant depends on the specific model of $V_s(r)$.

3. The equations obtained above can be used to analyze the bound and resonance levels of light nuclei.

As is well known, the nuclei ^{5}He and ^{5}Li do not form bound states, but appear as resonances with $l = 1$ in elastic scattering of a neutron and proton by a α particle. Their parameters[9] are presented in Table I. On nuclear scales, these resonances are situated near the threshold, so that Eq. (1) is applicable to them. For the neutral channel $n + \alpha$, setting $\zeta = 0$ (1) and $l = 1$, we obtain the well-known equation:

TABLE I.

Level	J^{π}, T	l	E_0, keV	Γ, keV	ζ	$1/A_l$	R_l	r_0/a_B
$^5He \to n+\alpha$	$\frac{3^-}{2}, \frac{1}{2}$	1	890	600 ± 20	0	− 994	− 37.9	0.16
$^5Li \to p+\alpha$	$\frac{3^-}{2}, \frac{1}{2}$	1	1970	~ 1500	− 2	− 1370	−11.8	–
$^8Li \to {}^7Li+n$	3^+, 1	1	228	31 ± 5	0	− 500	− 95	0.13
$^8B \to {}^7Be+p$	3^+, 1	1	2180	350 ± 40	−4	−2940	−49	–
$^7Li \to {}^3H+\alpha$	$\frac{7^-}{2}, \frac{1}{2}$	3	1166	93 ± 8	−2	-6.24×10^5	-4.17×10^4	0.75
$^7Be \to {}^3He+\alpha$	$\frac{7^-}{2}, \frac{1}{2}$	3	2980	175 ± 5	−4	-1.63×10^7	-4.26×10^5	1.5

$1/a_1(s) + 1/2r_1^{(s)}\lambda^2 + \lambda^3 = 0$. From here we easily determine $a_1^{(s)}$ and $r_1^{(s)}$, if we start from experimental data on the ^{5}He nucleus.

The results of the calculation are conveniently illustrated in the $(1/a_1^{(s)}, r_1^{(s)})$ plane (see Fig. 1). The straight line N gives a relation between the quantities $1/a_1^{(s)}$ and $r_1^{(s)}$, obtained from Eq. (5) from the $n+\alpha$ channel; the points correspond to different values of the width Γ. The curves C_1–C_3 are obtained from data on the charged channel $p+\alpha$: first, using Eq. (1) the parameters $a_1^{(cs)}$ and $r_1^{(cs)}$ (see Table I) were calcu-

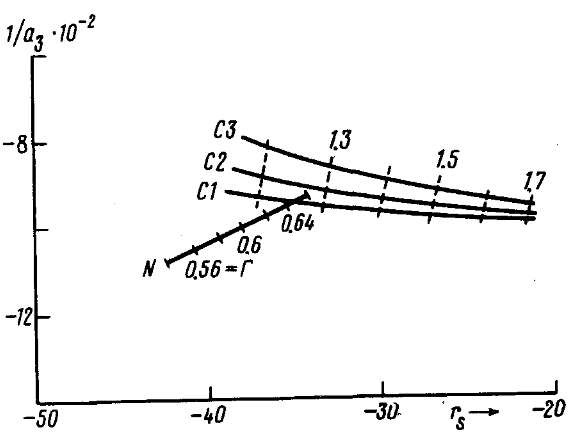

FIG. 1. The relation between the values of $1/a_1^{(s)}$ and $r_1^{(s)}$ extracted from the resonance energies and widths of levels in ^{5}He and ^{5}Li nuclei. The values of Γ (in MeV) are indicated for both levels. The unit of length is $L = 36$ fm.

lated; the Coulomb corrections were then introduced according to Eqs. (2) and (4). Here curve C_1 corresponds to a rectangular well, while curves C_2 and C_3 correspond to the Gaussian and Yamaguchi potentials. It is evident from Fig. 1 that the poor accuracy in determining the width of the resonance $\Gamma\,(^5\mathrm{Li}\to p+\alpha)$ does not allow calculating the parameters $a_1^{(s)}$ and uniquely. Nevertheless, the results in both calculations agree reasonably well with $\Gamma \approx 1.3$ MeV. For values of the parameters of the ^{5}He nucleus, presented in Table I, we obtain in the rectangular well model: $E_0 = 2.04$ MeV and $\Gamma = 1.24$ MeV, consistent with the experimental data for the ^{5}Li nucleus (the dependence of the values of E_0 and Γ, obtained in rescaling from the neutral to the charged channel, on the V_s model is insignificant). We note that in this case (in contrast to the s wave) the Coulomb interaction will strongly renormalize both low-energy parameters: $a_1^{(s)}$ and $r_1^{(s)}$. Thus, if we set $r_1^{(cs)} = r_1^{(s)}$ and use only the Coulomb correction (2) to the scattering length, then we obtain the following values for the energy levels in ^{5}Li instead of the values presented above: $E_0 = 1.47$ MeV and $\Gamma = 0.46$ which contradict experiment.

Similar calculations were performed for some other levels in isotopic multiplets: ^{8}Li and ^{8}B($l = 1$). ^{7}Li and ^{7}Be($l = 3$), and others. For these states $r_0/a_B < 1$. This ensures the applicability of Eq. (1), used to calculate a_l and r_l. From dimensional considerations, $a_l = A_l L^{2l+1}, r_l = R_l L^{1-2l}$, where $L = \hbar^2/me^2$ is the Coulomb unit of length ($a_B = L\,|\zeta\,|^{-1}$ is the Bohr radius), while A_l and R_l are dimensionless coefficients. The values of $1/A_l$ and R_l are presented in Table I.

We note that when calculating the Coulomb corrections, it is generally necessary to include the charge form factor of the nucleus in Eqs. (2)–(4). We shall examine this problem in a more detailed paper.

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[1)] Here $\chi_l(r) \sim r^{l+1}$ for $r\to 0$ and the normalization $\lim_{r\to\infty} r^l \chi_l(r) = 1$ is chosen. It is easy to see that the value of J_0 does not depend on the choice of the arbitrary parameter $R > 0$.

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