Use of final-energy sum rules to describe baryon properties in quantum chromodynamics and to estimate the proton lifetime in the SU(5) model

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Final-energy sum rules are used to calculate the proton coupling constant λ_p with an interpolating proton current. The coupling constant λ_p can be used to calculate the matrix elements for proton decay in the grand unified SU(5) model. The proton lifetime is calculated to be $\tau_p = 10^{28}\times(M_X/10^{14}\,\text{GeV})^4$ yr, where M_X is the mass of the X boson.

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Some experimental data just published show evidence of an instability of the proton; the lifetime of the proton estimated from these results is[1] $\tau_p \sim (5\text{–}7)\times 10^{30}$ yr. The grand unified models, as we know, predict an unstable proton.[2] To calculate the proton lifetime in the grand unified models is currently one of the most important problems of the physics of elementary particles, since a search for the decay of the proton is in fact the sole experimental test of the philosophy of grand unification. Furthermore, recent experimental data[1] have greatly intensified interest in the problem of the proton lifetime.

The Lagrangian describing the decay of the proton in the standard grand unified SU(5) model is,[2] if Cabibbo mixing is ignored (this approximation is eminently justified, since the Cabibbo angle is small, $\sin^2\theta_C = 1/25$),

$$\mathcal{L} = \frac{G_u}{\sqrt{2}}\xi\,\bar{e}^{\,c}_a(1+3\gamma_a)\,O_{1a}\ ,$$

$$O_{1a} = \epsilon^{ijk}u_k^T C\gamma_\mu(1-\gamma_5)u_j\,(\gamma^\mu d_i)_a\ , \qquad (1)$$

where $G_u/\sqrt{2} = g^2/8M^2$, g is the gauge coupling constant, and M is the mass of the X and Y bosons. The factor $\xi = 3.5$–4 allows for renormalization of the strong and electromagnetic interactions. The problem of calculating the proton lifetime in the SU(5) model (or in any grand unified model) is broken up into two steps. The first step is to determine the parameter $G_u/\sqrt{2}$ in effective Lagrangian (1). The second step is to evaluate the matrix elements for Lagrangian (1).

In this letter we calculate the matrix elements for proton decay through the use of a pole model (see Ref. 3, for example), working from final-energy sum rules. It should be noted that the matrix elements have been calculated previously in the nonrelativistic SU(6) model,[4] in the bag model,[5] and through the use of the technique developed by the team at the Institute of Theoretical and Experimental Physics.[3] Since the estimate of the proton lifetime is extremely important, we feel it worthwhile to derive some numerical values for the matrix elements of Lagrangian (1) by a new technique.

The two-point current function with the quantum numbers of the proton is[6,7]

$$\Pi(q) = i \int d^4x\, e^{iqx} \langle 0|\, TJ^{p'}(x)\bar{J}^{p}(0)|0\rangle$$

$$= \hat{q}\left[-\frac{(q^2)\ln -q^2/\mu^2}{8\pi(2\pi)^3} - \frac{2}{3}\langle 0|\,\bar{\psi}\psi\,|0\rangle^2 \frac{1}{q^2}\right]$$

$$+\frac{1}{(2\pi)^2}\langle 0\,|\,\bar{\psi}\psi\,|0\rangle q^2 \ln -\frac{q^2}{\mu^2} + \frac{1}{2(2\pi)^2} g_s \langle 0\,|\,\bar{\psi}\,\sigma_{\mu\nu}G^a_{\mu\nu}\frac{\lambda^a}{2}\psi\,|0\rangle \ln -\frac{q^2}{\mu^2}, \tag{2}$$

$$J^p(x) = u^a(x)C\gamma_\mu u^b\gamma_5\gamma_\mu d^c(x)\epsilon^{abc},$$

where C is the charge-conjugation matrix.

This expression is derived in the chiral limit, $m_u = m_d = m_s = 0$, and incorporates only a few of the leading terms in the expansion in $1/q^2$.

The function $\Pi(q)$ can be written in the Kallen-Lehmann representation:

$$\Pi(Q) = \int \frac{\hat{q}\rho_1(s) + \rho_2(s)}{s + Q^2}\, ds - \text{subtractions}, \quad Q^2 = -q^2. \tag{3}$$

Equation (3) yields sum rules by the standard technique[8–10]:

$$\int_0^{s_0} \rho_1^{\rm th}(s)ds = \int_0^{s_0} \rho_1^{\rm exp}(s)ds,$$

$$\int_0^{s_0} s\rho_1^{\rm th}(s)ds = \int_0^{s_0} s\rho_1^{\rm exp}(s)ds, \tag{4}$$

$$\int_0^{s_0} \rho_2^{\rm th}(s)ds = \int_0^{s_0} \rho_2^{\rm exp}(s)\, ds,$$

where the $\rho_{1,2}^{\mathrm{th}}(s)$ are given in approximation (2) by

$$\rho_1^{\mathrm{th}}(s) = \frac{s^2}{8\pi(2\pi)^3} + \frac{2}{3} <0|\bar{\psi}\psi|0>^2 \delta(s),$$

$$\rho_2^{\mathrm{th}}(s) = -\frac{s}{(2\pi)^2} <0|\bar{\psi}\psi|0> - \frac{1}{2(2\pi)^2} g_s <0|\bar{\psi}\sigma_{\mu\nu}\frac{\lambda^a}{2}G^a_{\mu\nu}\psi|0>. \quad (5)$$

Saturating the right side of sum rules (4) with a one-proton state, we find equations for the parameters s_0, M_p, and λ_p:

$$\lambda_p^2 = \frac{s_0^3}{24\pi(2\pi)^3} + \frac{2}{3} <0|\bar{\psi}\psi|0>^2,$$

$$\lambda_p^2 M_p^2 = \frac{s_0^4}{32\pi(2\pi)^3}, \quad (6)$$

$$\lambda_p^2 M_p = -\frac{s_0^2 <0|\bar{\psi}\psi|0>}{2(2\pi)^2} - \frac{s_0}{2(2\pi)^2} g_s <0|\bar{\psi}\sigma_{\mu\nu}\frac{\lambda^a}{2}G^a_{\mu\nu}\psi|0>,$$

where the coupling constant of the proton with the interpolating current, λ_p, is determined from

$$<0|J^p(0)|p(\mathbf{k},r)> = \lambda_p u_p(\mathbf{k},r). \quad (7)$$

A numerical value of the matrix element of the operator $\bar{\psi}\sigma_{\mu\nu}\frac{1}{2}\lambda^a\psi G^a_{\mu\nu}$ was found in Ref. 11; it is

$$g_s <0|\bar{\psi}\sigma_{\mu\nu}\frac{\lambda^a}{2}G^a_{\mu\nu}\psi|0> = m_0^2 <0|\bar{\psi}\psi|0>,$$

where $m_0^2 \sim 0.5$–$1\ \mathrm{GeV}^2$. For these numerical values, this operator has only a minor effect in Eqs. (6), and we will ignore it below. A solution of Eqs. (6) is

$$s_0 = 39.65\, |<0|\bar{\psi}\psi|0>|^{2/3} = 1.6 M_p^2,$$

$$\lambda_p^2 = 4 <0|\bar{\psi}\psi|0>^2 \doteq 0.26\times 10^{-3} M_p^6, \quad (8)$$

$$M_p = 4.98\, |<0|\bar{\psi}\psi|0>|^{1/3}.$$

From (7) and Lagrangian (1), which describes the decay of the proton in terms of quark fields, we find the effective Lagrangian

$$\mathcal{L}_{eff} = \frac{3G_u}{\sqrt{2}} \bar{e}_a^c J_a^p, \quad (9)$$

which describes the transition proton $\rightleftharpoons$ positron. We have calculated the constant λ_p already [Eq. (8)]; it is

$$\lambda_p = 1.8\times 10^{-4}\ \mathrm{GeV}^6. \quad (10)$$

Our result differs markedly from the result found by Ioffe[6] ($\lambda_p = 1.2\times10^{-3}$ GeV^6), which Beresinsky *et al.*[3] have used to evaluate the proton lifetime in the SU(5) model. By analogy with the case of mesons,[9,10] it seems extremely likely that the averaging interval s_0 in (4) should lie between the square of the proton mass and the square mass of the next radial excitation, $N(1470)$. We thus have a natural limitation on s_0: $M^2 \leqslant s_0 \leqslant M^2_{N(1470)}$. Alternately, using (6), we can write this limitation as

$$0.33\times10^{-4}\ \text{GeV}^6 \leqslant \lambda_p^2 7\times10^{-4}\ \text{GeV}^6. \tag{11}$$

The quantity $g^2/8M_x^2$ has been calculated by many investigators in the SU(5) model (see Ref. 2, for example). Most of the uncertainty in the estimate of $g^2/8M_x^2$ stems from the uncertainty in the choice of the scale for the strong interactions, $\Lambda_{\overline{MS}}$. Analysis of experimental data yields $\Lambda_{\overline{MS}} = 0.1 - 0.2$ GeV (Ref. 12). This value of $\Lambda_{\overline{MS}}$ leads to the following range for the mass of the X boson, where all the other uncertainties are also take into account:

$$M_X = (1\text{–}5)\times10^{14}\ \text{GeV}. \tag{12}$$

The quantity $a_s(M_X) = g^2/4\pi$ is known quite accurately[2]:

$$a_s(M_X) = 1/41.$$

In the pole model we have[3]

$$\Gamma(p\to e^+\pi^0) = \frac{5}{16\pi} G_u^2 \xi^2 \lambda_p^2 \frac{g_\pi^2}{m_p^2} = 0.5\Gamma_{tot}(p),$$

where g_π is the pion-nucleon coupling constant, which satisfies $g_\pi^2/4\pi = 14$. Taking the basic uncertainty, (12), and estimate (10) into account, we find, with $\xi^2 = 10$,

$$\tau_p = (M_X/10^{14}\ \text{GeV})^4 10^{28}\ \text{yr}.$$

With $M_X = 5\times10^{14}$ GeV we find $\tau_p \approx 6\times10^{30}$ yr, in agreement with the experimental results[1] on the lifetime of the proton lifetime: $\tau_p \sim (5\text{–}7)\times10^{30}$ yr.

To estimate an upper limit on the proton lifetime, we adopt $\lambda_p{}^2 = 0.33\times10^{-4}$ GeV^6, in accordance with inequality (11), and $M_X = 5\times10^{14}$ GeV; we find $\tau_p < 4\times10^{31}$ yr. [Beresinsky *et al.*[3] derived an upper limit $\tau_p < 2\times10^{30}$ yr, which, along with the recent experimental data, rules out the standard SU(5) model.]

Our estimate of the proton lifetime is similar to the estimates based on the nonrelativistic model, is seven times larger than the estimate of Ref. 3, and 3–14 times smaller than estimates based on the bag model.

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[1]K. Rubbia, Report to the Twenty-First International Conference on High Energy Physics, Paris, 26–31 July 1982.
[2]P. Langacker, Phys. Rep. 72C, 185 (1981); J. Ellis, "Phenomenology of Unified Gauge Theories," CERN Preprint TH-3174, 1981.

[3]V. Beresinsky, B. Ioffe, and Y. Kogan, Phys. Lett. **105B**, 33 (1981).
[4]J. Ellis *et al.*, Nucl. Phys. **B176**, 61 (1980); T. Goldman and D. A. Ross, Nucl. Phys. **B171**, 273 (1980); M. Machacek, *ibid.* **B159**, 37 (1979).
[5]A. Din, G. Girardi, and P. Sorba, Phys. Lett. **91B**, 77 (1980); J. F. Donoghue, *ibid.* **92B**, 99 (1980); E. Golowich, Phys. Rev. D **22**, 1148 (1980).
[6]B. L. Ioffe, Nucl. Phys. **B188**, 317 (1981).
[7]Y. Chung *et al.*, Phys. Lett. **102B**, 175 (1981).
[8]K. G. Chetyrkin, N. V. Krasnikov, and A. N. Tavkhelidze, Phys. Lett. **76B**, 83 (1978).
[9]N. V. Krasnikov and A. A. Pivovarov, Preprint NBI-HE-81-38, University of Copenhagen, 1981; Phys. Lett. **112B**, 397 (1982).
[10]N. V. Krasnikov, A. A. Pivovarov, and N. N. Tavkhelidze, Report to the International Seminar Kvarki-82 (Quarks—82), Sukhumi, May 1982.
[11]V. Novikov *et al.*, Proceedings of the International Conference Neutrino 78, Lafayette, 1978, p. 278.
[12]A. J. Buras, Rapporteür Talk at the 1981 Bonn Symposium on Lepton and Photon Interactions at High Energies; N. V. Krasnikov and A. A. Pivovarov, Phys. Lett. **115B**, 205 (1982).

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