Interaction of resonant particles with acoustic solitons in metals

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A theory is derived for acoustic solitons in a metal plate in a quantizing magnetic field with an electronic nonlinearity much greater than the lattice nonlinearity. The damping law for the soliton amplitude is found for collisionless and collisional damping.

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Let us examine the interaction of resonant electrons with acoustic solitons (compression pulses) in metals. We know that a necessary condition for the existence of an acoustic soliton is that the nonlinear and dispersive terms in the wave equation must be larger than the term describing the damping. It has been shown previously[1] that the nonlinear susceptibility in a quantizing magnetic field may be several orders of magnitude greater than the electronic nonlinear susceptibility in the absence of a field. A dispersion might result from (for example) the finite dimensions of the sample. We will therefore examine acoustic solitons in metal plates (or rods) in a quantizing magnetic field.[1)]

From the theory of elasticity we have the following equation for the fundamental symmetric mode propagating along the x axis for the case $d/L \ll 1$, where d is the plate thickness:

$$u_{tt} = c_0^2 u_{xx} + \gamma c_0^2 d^2 u_{xxxx} + \frac{\Lambda}{\rho}\frac{\partial n}{\partial x}. \tag{1}$$

Here

$$c_0 = \sqrt{\frac{E}{\rho(1-\sigma)^2}}, \qquad \gamma = \frac{\sigma^2}{12(1-\sigma)^2},$$

E is the Young's modulus, σ is the Poisson ratio, ρ is the density, and Λ is the strain energy. We find the contribution of the nonresonant particles to the equilibrium concentration which is quadratic in the wave amplitude from the following expression in the local approximation:

$$\delta n^{(2)} = \chi \Lambda^2 (u_x)^2$$

$$\chi = \frac{eH}{2\pi^2\hbar^2 v_c}\sum_{n=0}^{n_F}\int dp_x \frac{1}{2}\frac{\partial^2 F}{\partial \epsilon^2}, \qquad \chi_{max} = \frac{eH}{2\pi^2\hbar v_c}\frac{(2m)^{1/2}}{(kT)^{3/2}}, \tag{2}$$

where H is the magnetic field, e is the electron charge, v_c is the speed of light, the summation is over all the filled levels, p_x is the momentum projection on the magnetic field direction, and ϵ is the electron energy. Equation (2) describes an oscillatory dependence of the nonlinear susceptibility on the field with a maximum value χ_{max}.

Ostrovskiĭ and Sutin[3] have shown that expression (2) holds under the conditions $q^2/m \ll kT$, $mc_0^2 \ll kT$ when $\mathbf{q}\|\mathbf{H}$ or under the conditions $\omega\tau \ll 1$, $qR \ll 1$ when $\mathbf{q}\perp\mathbf{H}$, where R is the Larmor radius, m is the electron mass, and k is the Boltzmann constant.

Using (2), we can rewrite Eq. (1) as

$$u_{tt} = c_0^2 u_{xx} + \gamma c_0^2 d^2 u_{xxxx} + \frac{\Lambda^3}{\rho}\chi u_x u_{xx} + f(u), \tag{3}$$

where the functional $f(u)$ describes damping. In the absence of damping, Eq. (3) has a solution corresponding to a solitary wave—a compression pulse—

$$u(x,t) = u_0(\text{th}(x-ct)/L - 1). \tag{4}$$

Here $L = 12\gamma c_0^2 d^2\rho/\Lambda^3\chi u_0$ is the width of the soliton, and $c^2 = c_0^2(1+4\gamma d^2/L^2)$ is the wave velocity. How does the damping affect the soliton propagation? Assuming that the amplitude u_0 in (4) is a slowly varying function of the difference $(x-ct)$, we easily find the following equation from (3):

$$\frac{d\mathcal{H}}{dx} = -\rho\int_{-\infty}^{\infty} dx u_x f(u), \tag{5}$$

where $\mathscr{H}$ is the Hamiltonian of the system described by Eq. (3) without its last term. For solution (4) we have

$$\mathcal{H} = \frac{2\kappa}{3} u_0^3\left(1 + \frac{c^2}{c_0^2} + \frac{8}{3}\gamma\frac{u_0^2}{u_0^2(0)}\frac{d^2}{L^2}\right), \qquad \kappa = \frac{\Lambda^2\chi}{12\gamma d^2} . \tag{6}$$

Let us find the function $f(u)$ for various orientations of the magnetic field. For propagation along the field we have $f(u) = (\Lambda/\rho)(\partial n_{res}/\partial x)$, where n_{res} is the concentration of resonant electrons. We assume that the condition for the occurrence of giant quantum oscillations, $ql(\hbar\omega_c/\varepsilon_F)^{1/2} \gg 1$ $(q \propto L^{-1})$, does not hold, so that the interaction of a soliton with the resonant electrons can be treated by classical mechanics. We find n_{res} from the kinetic equation for the resonance electrons, which may be written as it would be in the absence of magnetic quantization, for the reason stated above.

The increment in the local equilibrium distribution function $g(v_x, x - ct)$ satisfies the equation

$$\frac{\partial g}{\partial t} + v_x\frac{\partial g}{\partial x} - \frac{\partial\Phi}{\partial x}\frac{\partial g}{\partial v_x} + \frac{g}{\tau_r} = -F_0'(\epsilon_\perp)\frac{\partial\Phi}{\partial t}, \tag{7}$$

where for solution (4) we have $\Phi(x - ct) = \Lambda\partial u/\partial x = -\Phi_0/\mathrm{ch}^2(x - ct)/L$, where $\Phi_0 = \Lambda u_0/L$, and $\epsilon_\perp$ is the energy of the motion in the plane perpendicular to H. Introducing the dimensionless variables $\xi = (x - ct)/L$, $s = (v_x - c)/\tilde{v}$ $\Psi = \Phi(\xi)/\Phi_0$ and $\tilde{v} = \sqrt{\Phi_0/m}$ and the parameter $a = L/\tilde{v}\tau_r$ (τ_r is the relaxation time), we can rewrite (7) as

$$s\frac{\partial g}{\partial\xi} - \frac{\partial\Psi}{\partial\xi}\frac{\partial g}{\partial s} + ag = \frac{F_0'(\epsilon_\perp)\Phi_0 c}{\tilde{v}}\frac{d\Psi}{d\xi} . \tag{8}$$

The solution of this equation in terms of the variables t and E can be written as

$$g(t, E) = \frac{\Phi_0 c}{\tilde{v}} F_0'(\epsilon_\perp)\int_{-\infty}^{t} e^{-a(t-\tau)}\frac{d\Psi}{d\xi}d\tau . \tag{9}$$

Here t is the time of the motion along the trajectory,

$$\frac{s^2}{2} - \frac{1}{\mathrm{ch}^2\xi} = E, \qquad s = \frac{d\xi}{dt}, \tag{10}$$

and E is the energy, which is negative for the particles trapped by the well and positive for the untrapped particles. Under nonlinear conditions ($a \ll 1$) the solutions for the distribution functions of the trapped and untrapped particles are

$$g_{tr} = F_0'(\epsilon_\perp)\frac{\Phi_0 c}{\tilde{v}}\{-s(t, E) + a\xi(t, E)\}, \qquad |s| \leqslant \frac{\sqrt{2}}{\mathrm{ch}\,\xi}, \tag{11}$$

$$g_{untr} = F_0'(\epsilon_\perp)\frac{\Phi_0 c}{\tilde{v}}\{-s(t, E) + s(-\infty, E) + a[\xi(t, E) - s(-\infty, E)t]\}, \quad |s| \geqslant \frac{\sqrt{2}}{\mathrm{ch}\xi} . \tag{12}$$

The function $g(s,\xi)$ determines the concentration of the resonant particles,

$$n_{\text{res}} = \frac{2}{(2\pi\hbar)^3}\int d\mathbf{p}\, g(v_x, x - ct, \epsilon_\perp). \tag{13}$$

Substituting $f(u) = (\Lambda/\rho)(\partial n_{\text{res}}/\partial x)$ into (5), we can write the right side of (6) as

$$-\Lambda \int_{-\infty}^{\infty} dx \frac{\partial u}{\partial x} \frac{\partial}{\partial x}\int g \frac{2d\mathbf{p}}{(2\pi\hbar)^3}. \tag{14}$$

Substituting the distribution functions for the trapped and untrapped particles, (11), into (14), we find

$$\frac{d\mathcal{H}}{dx} = -\frac{2\sqrt{2}}{3}\frac{a}{\pi\hbar^3}\Phi_0^2 m^2 c. \tag{15}$$

For the untrapped particles, the integral in (14) is evaluated in terms of the variables t, E, and $\epsilon_\perp$; for the trapped particles it is evaluated in terms of the variables ξ, s, and $\epsilon_\perp$. It turns out that the contribution of the untrapped particles to $d\mathcal{H}/dx$ is equal to that of the trapped particles. From (15) and (16) we can find the damping law for the soliton amplitude:

$$u_0(x) = u_0(0)\left(1 - \frac{x}{l}\right), \qquad l = \frac{\sqrt{2}\pi\hbar^3 \rho c}{m^2\Lambda^2}\cdot\tilde{v}(0)\tau_r\,. \tag{16}$$

It can be shown that in the x interval with $a \gtrsim 1$ the damping law in (16) becomes

$$u_0(x) = u_0(0)\,/\,(1 + x/l_B), \quad l_B = \frac{2\pi^2}{9\zeta(3)}\frac{\epsilon_F}{n_0}\frac{\rho c_0 v_F}{\Lambda^2}L(0) \tag{16a}$$

by analogy with Ref. 5. Here $\zeta(3)$ is the Riemann zeta function.

When the vector $\mathbf{H}$ is perpendicular to the plane of the plate, the damping is collisional, and the last term in (3) is given by[4]

$$f(u) = \frac{2}{15}\frac{n_0 m v_F^2}{\rho}\tau_r\frac{\partial^3 u}{\partial t\,\partial x^2}, \tag{17}$$

where n_0 is the equilibrium concentration, and v_F is the Fermi velocity. In this case we find the damping law for the soliton amplitude from (17), (6), and (5):

$$u(x) = u_0(0)\,/\,(1 + x/l_c)^{1/2}, \quad l_c^{-1} = \frac{2\pi}{15}\frac{n_0 m}{\rho c_0}\frac{v_F^2\tau_r}{L^2(0)}\,. \tag{18}$$

For some numerical estimates of the soliton parameters we adopt some typical values for a metal: $\Lambda \sim 10$ eV, $c_0 = 3\times 10^5$ cm/s at $T \sim 1$ K, $H \sim 10^4$ Oe, and $d \approx 0.03$ cm. For strains not exceeding 10^{-4} we find from (2a) and (4) $L \sim 0.01$ cm. According to (16) and (18), the decay length with $\tau_r \sim 10^{-8}$ s (we are assuming specular reflection) is much larger than the dimensions of the soliton.

[1)]Ostrovskiĭ and Sutin[3] studied acoustic solitons in plates and rods and showed that solitons can be formed over distances of 10^3 cm by virtue of the lattice nonlinearity. In this case it is difficult to arrange conditions such that the nonlinear term in the equation of elasticity is larger than the term responsible for the damping.

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