Transmission coefficient and the localization length of an electron in N bound disordered chains

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A model of N-coupled chains with defects for which it is possible to prove rigorously the presence of localization with arbitrarily strong coupling between chains is examined. N different localization lengths arise under these conditions.

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The exact solution of the problem of localization of single-electron states is known only for an elementary metallic chain,[1] when the localization length l^* is of the order of the free path l. On the other hand, there are strong arguments in favor of localization of all single-electron states in a metallic wire[2] and in the two-dimensional case when $l^* \gg l$ with weak disordering $p_F l \gg 1$. For this reason, it is interesting to examine this problem for N coupled chains.

As is well known,[5–8] the localization length enters explicitly into the expression for the decrease in the electron transmission coefficient with increasing dimensions L of the disordered region: $T \sim \exp(-L/l^*)$. Starting from this, in order to determine l^* we shall investigate the behavior of the transmission coefficient $T(L)$. This approach[5–8] to the problem turns out to be simpler than investigating the density-density correlation function.[1] It is assumed that the disordered segment $(0, L)$ consists of N chains, which can be represented as lying on a cylindrical surface parallel to its axis. The electron, together with the predominant motion along the chain, can hop in a perpendicular direction. Thus, in the system of functions $\{\psi_n(x)\}$, where $\psi_n(x)$ corresponds to the n-th chain, the Hamiltonian has the form

$$H_{nn'} = \left[-\frac{1}{2m}\frac{d^2}{dx^2} + U_n(x) \right] \delta_{nn'} + t(\delta_{n-1,n'} + \delta_{n+1,n'}). \tag{1}$$

Here $U_n(x)$ is a random impurity potential for the n-th chain and $t \ll E_F$ is the overlap integral describing electron hops to neighboring chains. In order to restrict the analysis to N chains, we shall supplement (1) with the following boundary condition:

$$\psi_N(x) = e^{i\alpha}\,\psi_0(x). \tag{2}$$

The phase factor $e^{i\alpha}$, which is responsible for the phase increase α with each revolution around the cylinder axis, destroys the symmetry with respect to time inversion ($\psi \to \psi^*$). This can be attributed to some magnetic flux inside the cylinder.

It is convenient to go over to a new system of wave functions:

$$\tilde{\psi}_n(x) = \frac{1}{\sqrt{N}} \sum_{n'=0}^{N-1} \exp(2\pi i n n'/N)\,\psi_{n'}(x), \tag{3}$$

$$\tilde{H}_{nn'} = \left[-\frac{1}{2m}\frac{d^2}{dx^2} + 2t\cos[(2\pi n + a)/N]\right]\delta_{nn} + \frac{1}{N}\sum_{n_0=0}^{N-1} U_{n_0}(x)\exp(2\pi i n_0(n-n')/N). \tag{4}$$

In the ordered regions, where $U_n(x) = 0$, it is possible to write immediately the solution of the Schrödinger equation in the form

$$\tilde{\psi}_n(x) = A_n \exp(ik_n x) + B_n \exp(-ik_n(x), \tag{5}$$

$$k_n = \sqrt{2mE - 2t v_F^{-1}\cos[(2\pi n + a)/N]}\,. \tag{6}$$

The constants A_n^L, B_n^L and A_n^R, B_n^R, which correspond to solution (5) on the left and right of the disordered section (0, L), are related by a linear transformation

$$\begin{bmatrix} A_n \\ B_n \end{bmatrix}^R = \hat{m} \begin{bmatrix} A_{n'} \\ B_{n'} \end{bmatrix}^L, \tag{7}$$

where $\hat{m}$ is a $2N$-dimensional complex matrix. The necessary information on scattering is contained in the matrix $\hat{M} = \hat{m}^+\hat{m}$, which was already introduced in Ref. 6 for a single chain. The condition for flux conservation imposes certain constraints on $\hat{m}$ and $\hat{M}$. We shall immediately write $\hat{M}$ in a form that satisfies these requirements:

$$\hat{M} = \begin{bmatrix} \hat{u}^+ \operatorname{ch}\hat{\Gamma}\,\hat{u} & \hat{u}^+ \operatorname{sh}\hat{\Gamma}\,\hat{v} \\ \hat{v}^+ \operatorname{sh}\hat{\Gamma}\,\hat{u} & \hat{v}^+ \operatorname{ch}\hat{\Gamma}\,\hat{v} \end{bmatrix}. \tag{8}$$

Here $\hat{u}$, $\hat{v}$ are unitary matrices, while $(\hat{\Gamma})_{nn'} = \Gamma_n \delta_{nn'}$ is a diagonal real matrix which determines the N transmission coefficients. In fact, if there is an incident flux $j_A^L = (A^+A)^L$ and a reflected flux $j_B^L = (B^+B)^L$ on the left of the disordered region, while on the right there is only a transmitted flux $j_A^R = (A^+A)^R$, then it is possible to express j_A^R in terms of the amplitude A_n^L of the incident flux:

$$j_A^R = (A^+)^L \hat{u}^+ \{2[\operatorname{ch}\hat{\Gamma} + 1]^{-1}\}\hat{u}(A)^L. \tag{9}$$

The diagonal matrix in braces in (9) contains N transmission coefficients $T_n = 2/(\operatorname{ch}\Gamma_n + 1)$.

In order to obtain a Fokker-Planck equation for the distribution function $W(L; \hat{\Gamma}, \hat{u}, \hat{v})$, it is necessary to follow the change in $W(L \ldots)$ with a small increase in the length L of the disordered section. We shall assume that the transmission amplitude through the separate impurity $d = \cos\gamma e^{i\delta}$ is close to unity: $\gamma \ll 1, \delta = 0$. Thus the scattering matrix $\hat{m}$ for an isolated impurity at the point x_0 of chain n_0 has the form:

$$\hat{m}(x_0 n_0) = 1 + \begin{bmatrix} 0 & \hat{\gamma}(x_0 n_0) \\ \hat{\gamma}^+(x_0 n_0) & 0 \end{bmatrix},$$

$$[\hat{\gamma}(x_0 n_0)]_{nn'} = -i\gamma N^{-1} \exp[-i(k_n + k_{n'})x_0 + 2\pi i n_0 (n - n')/N]. \tag{10}$$

Here we shall consider only the simplest case when, in deriving the Fokker-Planck equation, the averaging over the rapid dependences is performed with the help of the relation

$$\overline{\gamma_{nn'} \gamma^*_{mm'}} = \gamma^2 N^{-2} \delta_{nm} \delta_{n'm'}. \tag{11}$$

Using (6) and (10), we see that (11) is satisfied when N is an odd number, that there is no symmetry with respect to time inversion (i.e., α/π is not an integer) and that the following inequality is satisfied:

$$tl/v_F N^3 \gg 1.$$

Relation (11) is the key to simplifying the problem. In this case the distribution of $\hat{\Gamma}$ separates from the distributions of the matrices $\hat{u}$, $\hat{v}$. We note that this simplification does not occur for systems consisting of N chains lying on a surface and, in particular, for two chains.

The equation for the distribution function $W(L, \hat{F})$, where $\hat{F} = \mathrm{ch}\hat{\Gamma}$, (i.e., $T_n = 2/(F_n + 1)$ has the form

$$Nl \frac{\partial W}{\partial L} = \sum_{n=0}^{N-1} \frac{\partial}{\partial F_n} (F_n^2 - 1) \left\{ \frac{\partial}{\partial F_n} - 2 \sum_{n' \neq n} \frac{1}{F_n - F_{n'}} \right\} W(L, \hat{F}). \tag{12}$$

An analysis of Eq. (12) shows that it contains N different exponential dependences $F_n \sim \exp(L/l_n^*)$ and in addition the exponents differ generally by a quantity of the order of L/Nl. This means that at distances $L \gg Nl$ all F_n have different orders of magnitude. Let us renormalize F_n in increasing order. Thus for $L \gg Nl$ we have

$$F_0 \ll F_1 \ll \ldots \ll F_{N-1}. \tag{13}$$

The strong inequality (13) allows decoupling Eq. (12) into N equations for the distribution functions $W_n(L, F_n)$:

$$Nl \frac{\partial W_n}{\partial L} = \left\{ \frac{\partial}{\partial F_n} (F_n^2 - 1) \frac{\partial}{\partial F_n} - 2n \frac{\partial}{\partial F_n} F_n \right\} W_n(L, F_n), \tag{14}$$

where the number 1 next to F_n^2 should be retained only for $n = 0$, when (14) coincides with the equation for a single chain[5,8] to within the substitution $l \to Nl$.

An equation for N different localization lengths follows from (14) (l^* is the localization length in an isolated chain):

$$l_n^*(N) = Nl^*/(1+2n); \quad n = 0, 1, \ldots N-1. \tag{15}$$

They have the curious property that the average of their inverses is independent of N and is equal to $1/l^*$. The largest localization length $l_0^*(N) = Nl^*$ will make the greatest contribution to long-wavelength kinetic phenomena.

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