Cyclotron loss of fast particles from a bumpy tokamak

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A possible mechanism for the loss of fast particles from a tokamak is examined: a cyclotron resonance of the particles with the bumpiness perturbations of the toroidal magnetic field. The radial diffusion coefficient of the particles is estimated.

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The bumpiness of the toroidal magnetic field in a tokamak can substantially increase the transport coefficients in the plasma[1] and degrade the confinement of the fast charged products of fusion reactions.[2] In the design of tokamaks an effort is thus always made to reduce the bumpiness, in particular, by increasing the number of

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toroidal-winding coils. In this letter we show that if the number of toroidal magnet coils in a tokamak becomes large enough an additional mechanism for the loss of the fast particles may come into play. This mechanism involves a cyclotron interaction of the particles with the magnetic-field perturbations.

The bumpiness is a periodic perturbation of the magnetic field the long way around the torus, with a wave vector $k = N/R$, where N is the number of coils and R the major radius of the torus. If the particles have a high velocity, as they do if they are the products of fusion reactions, for example, and if the number of coils is large, then as a particle moves along a magnetic line of force, revolving in a Larmor circle at a frequency $\omega_i = eB/mc$, it can reach a resonance with the perturbations of the magnetic field,

$$k_{\parallel} v_{\parallel} = \pm \omega_i, \quad (1)$$

or

$$N|v_{\parallel}|/R = \omega_i. \quad (1a)$$

The effect of a cyclotron resonance on the motion of particles has been analyzed previously.[3] It was shown that with a pronounced bumpiness the effect can cause a stochastic conversion of passing particles into particles which are trapped at the local bumps of the magnetic field.

In this letter we take the nonuniformity of the tokamak magnetic field into account; the resonance is thus local in nature. As will be shown below, repeated crossing of the resonant point in the course of the motion of the particle can render the particle's path stochastic and thus cause a loss of fast particles.

The magnetic field B and the longitudinal particle velocity $v_{\parallel}$ in a tokamak vary along a line of force, so that condition (1) holds only at certain distinct resonant points on the line of force. Figure 1 shows phase trajectories of particles in a tokamak. As the coordinate along the line of force we have adopted the angle θ, reckoned along the minor azimuthal direction; the asterisks marke the resonant points. If we use the approximation $B = B_T R_0/R = B_T(1 - \epsilon \cos\theta)$ for the modulus of the unperturbed magnetic field B, we find from (1) that the particles which reach resonance are those for which the following holds:

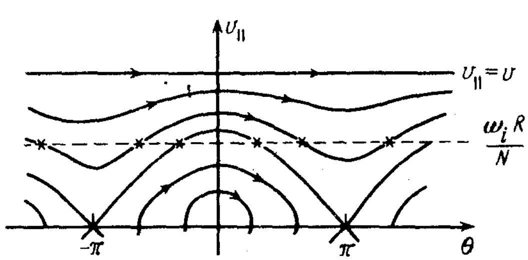

FIG. 1. Phase trajectories of particles in the $(v_{\parallel}, \theta)$ plane. The asterisks mark resonant points, with $N v_{\parallel}/R = \omega_i$.

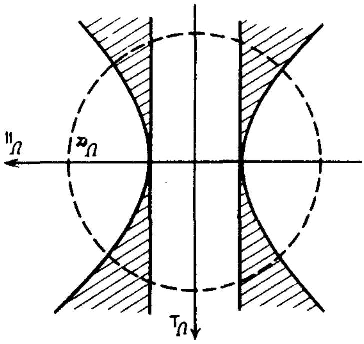

FIG. 2. Region of the resonant interaction in velocity space for $\theta = 0$ (the hatched region).

$$\frac{\omega_i^2 R^2}{N^2} < v_{\parallel 0}^2 < \frac{2\epsilon}{1+\epsilon} v^2 + \frac{\omega_i^2 R^2 (1-\epsilon)}{N^2 (1+\epsilon)} \ . \tag{2}$$

Here $v_{\parallel 0}$ is the longitudinal velocity of a particle on the outer side of the torus, with $\theta = 0$ and $\epsilon = r/R_0$. Figure 2 shows the resonant region in velocity space for $\theta = 0$.

At resonance (1) the magnetic moment of the particle ceases to be conserved, and the magnetic moment acquires a finite increment in successive crossings of the resonant points. The energy of the particle, of course, does not change. The increment in the magnetic moment can be calculated by projecting the equation of motion $\mathbf{v} = (e/mc)[\mathbf{v}\times\mathbf{B}]$ on the direction of a magnetic line of force:

$$v_{\parallel} = -\frac{v_{\perp}^2}{2B}(\mathbf{b}\cdot\vec{\nabla})B + \frac{v_{\parallel} v_{\perp}}{B}\{(\mathbf{e}_1\cdot\vec{\nabla})B\sin\alpha + (\mathbf{e}_2\cdot\vec{\nabla})B\cos\alpha\} + \{\ \cdots\ \} \ . \tag{3}$$

Here $\mathbf{b} = \mathbf{B}/B$; $\mathbf{v} = \mathbf{v}_{\parallel}\mathbf{b} + \mathbf{v}_{\perp}\mathbf{e}_1\sin\alpha + \mathbf{v}_{\perp}\mathbf{e}_2\cos\alpha$, $\mathbf{e}_1$ is a unit vector normal to the magnetic surface, given by $\mathbf{e}_1 = \nabla\psi/|\nabla\psi|$, and $\mathbf{e}_2 = [\mathbf{b}\times\mathbf{e}_1]$. If the magnetic surface, given by $B = B_{\mathrm{T}}(1 - \epsilon\cos\theta + \delta\sin N\phi)$, where ϕ is the major azimuthal angle of the torus, then after taking an average over the unperturbed particle motion ($\dot{\alpha} = \omega_i(\theta), \dot{\phi} = v_{\parallel}/R, \mathbf{r}(t) = \mathbf{r}_c + (1/\omega_i)[\mathbf{b}\times\mathbf{v}]$) we find that the second term in (3) causes a jump in $v_{\parallel 0}$ at resonant point (1a):

$$\Delta v_{\parallel 0} \simeq \sqrt{\frac{\pi N q}{\epsilon}}\,\delta\cos\left(N\phi_* - \alpha_* - \frac{\pi}{4}\right) \ . \tag{4}$$

Contributing to the time average in (4) is a narrow region of width $\Delta\theta \approx (\epsilon q N)^{-1/2} \ll 1$ near the resonant point. The subscript asterisk denotes the value of ϕ or α at the resonant point. The magnitude of the velocity increment depends on the phase $\Phi = N\phi_* - \alpha_* - \pi/4$ with which the particle arrives at the resonant point, so that the particles will trace out some rather complicated trajectories in the phase plane. Numberical calculations will be required to analyze these trajectories. We restrict the present discussion to a rough estimate of the necessary condition for the onset of stochastic trajectories. According to Ref. 4, this condition can be written

$$\left|\Delta v_{\parallel 0}\frac{\partial}{\partial v_{\parallel 0}}[\Delta\Phi(v_{\parallel 0})]\right| > 1, \tag{5}$$

where $\Delta\Phi$ is the phase shift between successive crossings of the resonant point by a particle, given by

$$\Delta\Phi = \int_{t_1}^{t_2} (N\dot{\phi} - \dot{\alpha})dt \approx 2q\epsilon N v_{\perp 0}^2 \int_0^{\theta_*} \frac{(\cos\theta - \cos\theta_*)d\theta}{v_{\|}^2 + v_{\|}v_{\|0}} . \tag{6}$$

Using (4) and (6), we can write condition (5) as

$$\delta > (\pi\epsilon N^3 q^3)^{-1/2} \tag{7a}$$

for passing particles and

$$\delta > (\epsilon/\pi N^3 q^3)^{1/2} \tag{7b}$$

for trapped particles. For the parameters of the planned T-14 device[5] ($N = 32, R_0 = 41$ cm, $B = 14$ T), for example, the DT α particles reach resonance at $v_{\|}/v \simeq 0.5$. From (7a) and (7b) we find $\delta > 10^{-3}$ and $\delta > 3\times 10^{-4}$, respectively.

Diffusion in velocity space, which should occur under condition (7a) or (7b), will be accompanied by a radial diffusion of the fast particles. The diffusion coefficient D_r can be estimated from the condition that the average generalized angular momentum of the particle, $mv_{\|}R + e\psi/2\pi c$, is conserved between successive events in which a particle is scattered at a resonant point:

$$D_r \simeq \frac{v_{\|}}{qR}\Delta r^2 \approx \frac{v_{\|}}{qR}\left(\frac{mcq}{eB\epsilon}\Delta v_{\|0}\right)^2 .$$

During the time spent by the particle in the resonant zone, $t \sim (\epsilon^2 v^2/\Delta v_{\|0}^2)(qR/v_{\|})$, the particle manages to undergo a radial displacement

$$\Delta a \simeq qa/\epsilon N,$$

which is comparable to the radius of the plasma column. If $\Delta a \ll a$, particle loss can result only from scattering into the loss cone.

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