Scattering length in a system of three charged particles

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A modified definition is offered for the scattering length in a system of three charged particles. A correct method for calculating it is described. This method is used to calculate the scattering length for the scattering of the proton by the deuteron.

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1. The scattering length is a fundamental characteristic of nuclear collisions. From the results determined on scattering lengths one can draw conclusions about various parameters of nucleon-nucleon interactions, e.g., the validity of the hypothesized isotopic invariance of nuclear forces.[1]

There is considerable interest in calculating the scattering lengths in few-nucleon systems by methods that can guarantee results of a specified accuracy. To date, however, such calculations have been carried out only for systems of neutral particles. In particular, the *n-d* scattering lengths have been calculated for realistic nuclear potentials (see Refs. 2–4, for example).

The scattering lengths in systems of charged particles have not previously been calculated correctly, because no numerical method has been available for solving the scattering problem of three Coulomb particles on the basis of the initial dynamic formulation of the problem. Recently, however, substantial progress has been made toward the solution of this problem. A correct calculation method for dealing with the scattering of three charged particles was developed in Ref. 5, and a definition of the scattering length in such systems was given in Ref. 6. As a result, it has become possible to proceed to calculations of the scattering lengths for three-particle systems with a long-range Coulomb interaction.

In the present letter we describe a method for calculating the scattering lengths in a system of three charged particles on the basis of a differential formulation of the scattering problem.[7] We will use this method to calculate *p-d* scattering lengths.

2. We consider a system of three charged particles that interact through potentials $V_\alpha(x_\alpha)$ $(\alpha = 1,2,3)$, which contain short-range nuclear parts along with a purely Coulomb part. We assume that the corresponding two-particle binary Hamiltonians $h_\alpha = -\Delta + V_\alpha$ have bound states $\psi_{\alpha,i}(x_\alpha)$ with energies $-\varepsilon_{\alpha,i}$ $(i = 1,2,\ldots, N_\alpha; N_\alpha \leqslant \infty)$.

The elastic amplitude $f_{\alpha,i}(p'_\alpha, p_\alpha)$ in such a system for an energy $E = p_\alpha^2 - \varepsilon_{\alpha,i}$ can be written[6]

$$f_{\alpha,i}(p'_\alpha, p_\alpha) = f_0(p'_\alpha, p_\alpha) + \chi_s(p_\alpha^2) f_{\alpha,i}(p'_\alpha, p_\alpha), \tag{1}$$

where the functions f_0 and χ_s contain all the singularities in the elastic amplitude for the threshold energy $E = -\varepsilon_{\alpha,i}$. These singularities are consequences of the long-

range part of the binary potentials. We wish to emphasize that the singularity of the function f_0 is not a purely Coulombic singularity; it instead contains some additional singularities which correspond to the multipole part of the interaction of the incident particle with the effective target potential $V_{\alpha,i}$. This potential is asymptotically equal to the sum of the Coulomb and multipole parts:

$$V_{\alpha,i}(y_\alpha) \sim \frac{n_\alpha}{|y_\alpha|} + \sum_{l=1}^{\infty} \frac{\mu_l(\hat{y}_\alpha)}{|y_\alpha|^{l+1}},$$

where n_α is the total charge of the target, and μ_l are the multipole moments of pair α, averaged over the wave function $\psi_{\alpha,i}$. The singularities of the function f_0 are given explicitly in Ref. 6.

The second term in (1) corresponds to the nuclear part of the interaction. Its singularity at $p_\alpha^2 = 0$ is described by the factor χ_s:

$$\chi_s(p_\alpha^2) = |p_\alpha|^{-1} \exp\{2i\eta_\alpha(\ln|\eta_\alpha| - 1) + 2\pi\eta_\alpha\varepsilon(n_\alpha)\},$$

where $\eta_\alpha = n_\alpha/2|p_\alpha|$, and ε is the Heaviside unit step function [$\varepsilon(n) = 0$ at $n<0$, $\varepsilon(n) = 1$ at $n>0$]. The function $f_{\alpha,i}$ has a finite limit at the threshold energy, which can naturally be called the "modified scattering length" $a_{\alpha,i}$ in the given system:

$$a_{\alpha,i}(\hat{p}_\alpha', \hat{p}_\alpha) = \lim_{p_\alpha^2 \to 0} \tilde{f}_{\alpha,i}(p_\alpha', p_\alpha).$$

For spherically symmetric target states $\psi_{\alpha,i}$, we might note, the multipole part of the potential $V_{\alpha,i}$ vanishes, and the singularity of f_0 is purely Coulombic. As a result, the definition of the scattering length offered above agrees with this ordinary definition for charged particles in this case.[8]

The problem of determining the scattering length thus reduces in practice to one of determining the singular terms of the three-particle elastic amplitude; after these singular terms are subtracted, the remainder is in the factorized form $\chi_s\tilde{f}$, where $\tilde{f}$ is a smooth, bounded function. The latter function corresponds to the nuclear part of the interaction.

To illustrate the method for calculating the scattering length by the approach described above, we consider the scattering of a proton by a deuteron. We specify the nuclear interaction by means of MT I–III potentials, from Ref. 9.

To solve the *p-d* scattering problem we use a very simple version of the modified Faddeev differential equations, in which the entire Coulomb interaction is incorporated in the unperturbed Hamiltonian. After the components of the wave function are expanded in spherical harmonics,[10] we find a system of integrodifferential equations for the partial components.[5] The numerical method used to solve these equations[5] is based on a finite-difference approximation and is a generalization of the method developed in Ref. 10.

This method can be used to calculate the amplitude for elastic scattering at several energies near the threshold. Then subtacting from the elastic amplitude the function f_0, and multiplying the remainder by χ_s^{-1}, we find the energy dependence of the

function $f_{\alpha,i}$ near the threshold. The scattering length is then found by extrapolating this dependence to the point $E = -\varepsilon_{\alpha,i}$.

As a result, the doublet (total spin $S = 1/2$) and quadruplet ($S = 3/2$) p-d scattering lengths ${}^{2S+1}a$ are written as the partial series

$$ {}^{2S+1}a(\hat{p}', \hat{p}) = \frac{1}{a_c} \sum_{L=0}^{\infty} a_L^S P_L(t), \tag{2} $$

where $t = (\hat{p}', p)$ and the normalization factor a_c is the Coulomb radius of the p-d system. The summation in (2) is over all values of the total orbital angular momentum. The coefficients a_L^S are the scattering lengths with a fixed total orbital angular momentum. The calculations show that series (2) converges extremely rapidly, and all terms with $L \geqslant 1$ can be ignored in calculations of the scattering lengths. Consequently, ${}^{2S+1}a$ is very nearly equal to the scattering length in the S wave.

For the scattering lengths we find the following results:

$$ {}^2a = 1{,}03\,\text{fm}, \qquad {}^4a = 11{,}96\,\text{fm}. $$

For comparison, the experimental values of these lengths are[11,12]

$$ {}^{11}: \ {}^2a = 1{,}3 - 0{,}2\,\text{fm}, \qquad {}^4a = 11{,}4^{+\,1{,}8}_{-\,1{,}2}\,\text{fm}, $$

$$ {}^{12}: \ {}^2a = 2{,}73 - 0{,}1\,\text{fm}, \qquad {}^4a = 11{,}88^{+0{,}4}_{-0{,}1}\,\text{fm}. $$

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