Stimulated scattering of sound by an acoustic flow of a viscous liquid

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A new type of stimulated scattering of acoustic waves in absorbing liquids is proposed. The gain is calculated for cw and pulsed operation. The effect should be observable in experiments in viscous liquids such as benzene at an ultrasonic power on the order of 10^{-1}–10 W.

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1. Stimulated scattering of sound has not previously been studied experimentally in homogeneous liquids. Two mechanisms for stimulated scattering have been analyzed theoretically: scattering by acoustically induced vortex flows[1] and scattering by temperature waves.[2] It is quite reasonable to examine the possibility of using stimulated scattering for conversions of narrow-band acoustic signals and for measurements of the kinetic properties of liquids. The primary difficulty in observing stimulated scattering in the ultrasonic range is the pronounced elastic nonlinearity, which causes a severe energy loss in the cascade generation of harmonics. The only realistic mechanisms for stimulated scattering, therefore, are those involving scattering by excitations of a nonacoustic nature which provide a sufficient amplification of a signal wave over distances shorter than the shock formation length at a feasible acoustic power level. This requirement can be satisfied by a universal mechanism for stimulated scattering by an acoustic flow, which we will outline in this letter. In this mechanism the scattering lattice is formed by virtue of a hydrodynamic nonlinearity: The local flow of a viscous liquid is directed away from antinodes toward nodes of a quasistanding acoustic wave (another possible manifestation of this nonlinearity is a self-focusing of narrow acoustic beams[3,4]). Since this effect is quite universal (the governing parameters are the density, the sound velocity, and the viscosity), it may also be applied interest, in that it limits the depth to which intense sound can penetrate into thick layers of liquids.

2. Analysis of the system of hydrodynamic equations in which heating of the liquid is ignored shows that a collinear geometry is the most effective for stimulated

scattering by an acoustic flow (the directional pattern is pointed backward). In this geometry the acoustic pressure p and the flow velocity V satisfy the equations

$$[c^2\partial^2/\partial x^2 - \partial^2/\partial t^2 + \frac{4}{3}c\nu\partial^3/\partial x^3]p = 2V\partial^2 p/\partial x\,\partial t,$$
$$(\partial/\partial t - \nu\partial^2/\partial x^2)V = -(2\rho^2 c^3)^{-1}(c\partial/\partial x + \partial/\partial t)p^2. \quad (1)$$

Here ρ is the density, c is the sound velocity, $\nu = \eta/\rho$, and η is the shear viscosity. We seek a solution of (1) in the form

$$p = (p_p/2)\exp[i(k_p x - \omega_p t)] + (p_s/2)\exp[-i(k_s x + \omega_s t)] + \text{c.c.}, \quad (2)$$
$$V = (p/\rho c) + (u/2)\exp[i(qx - \Omega t)] + \text{c.c.}$$

The approximately steady-state pressure amplitude satisfies $p(\Omega \ll 2ck) \propto u^2(\Omega \ll 2ck)$, i.e., is a small quantity of second order. The frequencies and wave numbers of the pump (the amplitude p_p) and of the scattered acoustic wave (p_s) satisfy $k_p + k_s = q$, $\omega_p - \omega_s = \Omega$.

In the steady state [$\tau > L/c, (\nu q^2)^{-1}$; τ is the duration of the pump pulse, and L is the length of the medium] the truncated equations for the amplitudes from (2) have the solution $|p_s| = \exp G p_s(x = L)$ with the gain G given by

$$G/\delta L = (I/I_{th})[1 + (\Omega/\Delta\Omega)^2]^{-1}[1 - \exp(-2\delta L)]/2\delta L - 1.$$

Here $\delta = \frac{2}{3}\nu\omega^2/c^2$ is the absorption coefficient, $I = p_p^2(x = 0)/2\rho c$ is the pump intensity at the entrance to the medium, and $\Delta\Omega = \nu q^2$ is the line width of the stimulated scattering. A typical estimate (for benzene) yields $\Delta\Omega/2\pi = 10$ Hz at the frequency $\omega_p/2\pi = 1$ MHz (here and below, we are using the numerical values from Ref. 5 for the parameters). The quantity $I_{\text{th}} = 4\nu\rho\delta c^2$ is a threshold pump intensity [$G(I > I_{\text{th}}) > 0$] for scattering without a frequency shift ($\Omega = 0$) in a liquid which absorbs only slightly ($\delta L \ll 1$). The threshold condition can definitely be met in liquids with a moderate viscosity; for benzene and acetone, the threshold intensities are 0.40 and 0.014 W/cm^2.

3. Many liquids have to be ruled out for observing stimulated scattering by an acoustic flow because of competing nonlinear effects. The predominant process in very viscous liquids is the stimulated thermal scattering which results from the temperature dependence of the sound velocity ($\gamma = \partial \ln c^2/\partial T \neq 0$). A calculation of the efficiency of this scattering,[2] from Eqs. (1) supplemented by the heat-conduction equation, shows that the threshold for stimulated thermal scattering (with the classical absorption mechanism) is proportional to ω, while the threshold for stimulated scattering by an acoustic flow is proportional to ω^2. The condition under which the latter scattering can be distinguished thus reduces to a condition on the frequency: $\omega < \omega_T = 3\kappa\rho/|\gamma|\eta^2$, where κ is the thermal conductivity. For glycerin, for example, we would have $\omega_T/2\pi \cong 5 \times 10^4$ Hz; in benzene, on the other hand, stimulated scattering by an acoustic flow will be predominant at frequencies up to 7×10^{10} Hz.

In flowing liquids, on the other hand, a shock wave arises quite rapidly. A time-varying generalization of the theory for stimulated scattering by an acoustic flow yields the growth rate which is required in comparison with the reciprocal of the shock

formation length $L_b^{-1} = \omega\epsilon p_p(x=0)/\rho c^3$ (ϵ is the elastic nonlinearity coefficient) at a pump level well above the threshold. Under the condition $\tau > Lc^{-1}$ we can calculate the maximum growth rate $G(t_r)$ and the delay of the scattered pulse, t_r. The asymptotic values of these quantities at $I/I_{\text{th}} > 1$ and $\delta L < 1$ are

$$t_r = \delta L c^2 I / 2\omega^2 \nu I_{th}; \qquad G(t_r) = \delta L (I/I_{th} - 1) - 0.5 \ln(4\pi\delta L T / I_{th}). \tag{3}$$

We require that the gain over the distance $L = L_b/2$ be 10: $G(t_r, L_b/2) = 2.3$. It follows from (3) that for the frequency $\omega/2\pi = 1$ MHz the pump intensities in benzene and acetone would have to be 90 and 35 W/cm^2; at these pump intensities we would have $t_r = 0.13$ and 0.17 s and $L_b = 4.3$ and 4.8 cm. At lower frequencies, the pump need not be as intense, since we have $I_{\text{th}} \sim \omega^2$, while $L_b \sim \omega^{-1}$. For the frequency $\omega/2\pi = 10^2$ kHz we find $I(G = 2.3) = 1$ and 0.6 W/cm^2. The requirements for the spatial and temporal parameters, on the other hand, are more stringent: $t_r = 14$ and 29 s and $L_b = 410$ and 480 cm. The inequality $\delta L < 1$ holds in all cases.

We can estimate the increase in the temperature of the medium from the heat-conduction equation: $\Delta T \cong 2\delta I t_r/\rho c_p$, where c_p is the specific heat. For the parameter values used here we find $\Delta T < 1$ K. The time required to establish a steady-state transverse convective flow in a beam of width $a \cong 1$ cm is $\tau_s \cong a^2/\nu \gtrsim 10^2$ s, i.e., longer than t_r, so that the velocity of this flow can be estimated to be $V_c \cong \Delta T \alpha g t_r$ (α is the thermal expansion coefficient, and g is the acceleration due to gravity). The time-varying analog of the Grashof condition, $V_c t_r < a$, is satisfied by a margin of an order of magnitude under the conditions assumed here.

In summary, stimulated scattered by an acoustic flow in a liquid such as benzene can be observed experimentally at a pump power level on the order of 10^{-1}–10 W in the frequency range 0.1–1 MHz.

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