Quantum corrections to the surface conductivity of a disordered metal

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(Submitted 2 November 1982)
Pis'ma Zh. Eksp. Teor. Fiz. **36**, No. 11, 394–396 (5 December 1982)

The elastic reflection of electrons from a bulk metal target intensifies interference effects and gives rise to a pseudo-two-dimensional quantum correction to the surface conductivity, $\Delta\sigma_s \sim \ln T$. A logarithmic contribution of $\Delta\sigma_s$ to the zero anomaly of the tunneling resistance is discussed.

PACS numbers: 73.25. + i

Localized quantum corrections to the conductivity, $\Delta\sigma$, result from an interference of electron waves which are propagating along a common path in opposite directions.[1] The interference raises the probability for an electron to return to the starting point and thus reduces σ. The effect intensifies as the dimensionality of the space is lowered: As the temperature T is reduced, $\Delta\sigma(T)$ exhibits a plateau in the three-dimensional ($3D$) case but exhibits an $\ln T$ in the $2D$ case. This divergence occurs in a film with a thickness L_z small in comparison with the relaxation length (L_φ) of the phase of the wave function.[2]

In this letter we show that even in thick films ($L_z \gg L_\varphi$) the correction $\Delta\sigma$ contains, in addition to the $3D$ component, a surface component which simulates a $2D$ correction. It follows, in particular, that the experimental dependence $\Delta\sigma \sim \ln T$ in films cannot be taken as unambiguous evidence that the samples are of a $2D$ nature.

We consider a diffusive motion of an electron in a layer of thickness $L_\varphi/2$ near the surface $z=0$. If reflection from this surface does not disrupt the phase, then the probability for the electron to return to its starting point, $\mathbf{r}' = (x', y', z')$, increases to a greater extent as $\mathbf{r}'$ is moved closer to the surface. Solution of the diffusion equation to find the return probability yields an additional term $\sim 1/z'$ due to the "image" at the point $(x', y', -z')$. The additional correction to the local conductivity is thus $\sim 1/z'$, while that to the surface conductivity is $\sim \int 1/z' dz' \simeq \ln L_\varphi / l \sim \ln T$ (l is the mean free path, and $L_\varphi \sim T^{-p}$).

For a quantitative description of the effect we use the method of Refs. 3 and 4. The local correction to σ at the frequency, ω in a magnetic field $\mathbf{H}$,

$$\Delta\sigma(\mathbf{r}, \omega) = -\frac{2e^2 D}{\pi\hbar} C(\mathbf{r}, \mathbf{r}, \omega) , \tag{1}$$

is determined by the solution of the equation

$$\left[-i\omega + D\left(-i\nabla - \frac{2e}{\hbar c} \mathbf{A} \right)^2 + \tau_\varphi^{-1} \right] C(\mathbf{r}, \mathbf{r}', \omega) = \delta(\mathbf{r} - \mathbf{r}') , \tag{2}$$

with the following boundary condition[5] at $z=0$:

$$\left(\nabla_z - \frac{2ie}{\hbar c} A_z \right) C = 0, \tag{3}$$

where D is the diffusion coefficient, $\tau_\varphi = L_\varphi^2 / D$, and $\text{rot}\mathbf{A} = \mathbf{H}$ (rot = curl).

1. Let us asume a semi-infinite sample: $z \geqslant 0$, $H = \omega = 0$. We write the solution of Eqs. (2) and (3),

$$C(z, z) = \sum_{\mathbf{q}}^{q_z \geqslant 0} B^2(q_z) \cos^2 q_z z (Dq^2 + \tau_\varphi^{-1})^{-1} = (2\pi)^{-3} \int d^3 q \; 2\cos^2 q_z z (Dq^2 + \tau_\varphi^{-1})^{-1} , \tag{4}$$

as the sum of a volume term C^{3D} and a surface term $\tilde{C}$:

$$C^{3D}(z, z) = (2\pi)^{-3} \int d^3 q (Dq^2 + \tau_\varphi^{-1})^{-1} = (4\pi D)^{-1} (2/\pi l - 1/L_\varphi) , \tag{5}$$

$$\tilde{C}(z, z) = (2\pi)^{-3} \int d^3 q \cos 2 q_z z (Dq^2 + \tau_\varphi^{-1})^{-1} = (8\pi D z)^{-1} \exp(-2z/L_\varphi) . \tag{6}$$

The integration is now over all $\mathbf{q}$, and we have $B(q_z) = \sqrt{2}$ at $q_z > 0$ and $B(0) = 1$. Substituting $C = C^{3D} + \tilde{C}$ into (1), and integrating over z, we find the correction to the surface conductivity to be

$$\Delta\sigma_s \equiv \int_l^\infty \Delta\sigma(z)\,dz = L_z\Delta\sigma^{3D} + \frac{1}{4}\Delta\sigma^{2D}, \tag{7}$$

where L_z is the normalization thickness of the sample, and $\Delta\sigma^{3D} = -(2/\pi l - 1/L_\varphi)/2\pi^2$ and $\Delta\sigma^{2D} = -\pi^{-2}\ln L_\varphi/l$ are the known[1] $3D$ and $2D$ corrections to σ, expressed in units of $e^2/\hbar$. Similarily, for a semi-infinite sample of any dimensionality we find

$$\Delta\sigma_s^d = L_z\Delta\sigma^d + \frac{1}{4}\Delta\sigma^{d-1}, \qquad d = 1D,\ 2D,\ 3D. \tag{8}$$

The first term in (7) and (8) is the volume component, while the second is the surface component.

2. We now assume a film of thickness $L_z \gg l$, with $H = \omega = 0$. If $L_z \gg L_\varphi$, the contributions from the two surfaces are summed, and the coefficient of $\Delta\sigma^{2D}$ in Eq. (7) is doubled. In general, the expression for C [see Eq. (4); $q_z = \pi n/L_z$, $n = 0, 1, 2, \ldots$] reduces to

$$C(z,z) = (4\pi D L_z)^{-1}\left\{\ln\left(\frac{L_\varphi \operatorname{sh} L_z/l}{l \operatorname{sh} L_z/L_\varphi}\right) + \int_{L_z/L_\varphi}^{L_z/l} dt\left[\frac{\operatorname{ch} t(1 - 2z/L_z)}{\operatorname{sh} t} - \frac{1}{t}\right]\right\}. \tag{9}$$

Integrating over z, we find, for a square film,

$$\Delta\sigma_s = -\frac{e^2}{2\pi^2\hbar}\left[\ln\frac{L_\varphi}{l} + \ln\left(\frac{\operatorname{sh} L_z/l}{\operatorname{sh} L_z/L_\varphi}\right)\right] \tag{10}$$

In thin films $(L_z \ll L_\varphi)$ we find from (10)

$$\Delta\sigma_s = -\frac{e^2}{\pi^2\hbar}\ln\left(\gamma\frac{L_\varphi}{l}\right), \tag{11}$$

which differs from the ordinary $\Delta\sigma^{2D}$ by the factor γ, which does not alter the dependence $\Delta\sigma_s \sim \ln T$; $\gamma^2 = (l/L_z)\operatorname{sh} L_z/l$.

3. We now assume that a magnetic field is imposed in the direction perpendicular to the surface, and we have $\omega = 0$. For a semi-infinite sample the surface magnetoconductivity $\delta\sigma_s = \sigma_s(H) - \sigma_s(0)$ is

$$\delta\sigma_s(H) = L_z\delta\sigma^{3D}(H) + \frac{1}{4}\delta\sigma^{2D}(H), \tag{12}$$

and $\delta\sigma^d(H)$ is the magnetoconductivity in the d-dimensional case.[3,4,6]

For a thick film $(L_z \gg L_\varphi)$ the second term in (12) is doubled. In a thin film $(L_z \ll L_\varphi)$ the result depends strongly on the ratio of $L_H^2 \equiv \hbar c/2eH$ and L_z^2: $\delta\sigma_s(H)$ behaves in the $2D$ manner only at $L_H \gg L_z$, while in strong fields $(L_H \ll L_z)$ the $3D$ component becomes predominant.

Similar corrections to $\Delta\sigma_s$ should arise in an interaction theory which incorporates an interference of electron-electron and electron-impurity scattering,[3,4] Incorporation of spin scattering, intervalley scattering, etc., in the standard way[4] changes the coefficients in $\Delta\sigma^d$.

The corrections $\Delta\sigma_s$ should be seen in effects which are sensitive to surfaces, e.g., the skin effect, contact phenomena, etc. We will briefly discuss the problem of the tunneling anomaly at a bias voltage $V\simeq 0$, seen as a dip in the tunneling conductivity[7] $\sigma_T(V)$. In addition to the component $\Delta\sigma_T \sim \sqrt{V}$ due to the change in the volume state density at the Fermi level, due in turn to the interaction,[8] we can expect a component from surfaces adjacent to the tunneling gap. This component would have the behavior $\Delta\sigma_T \sim \ln T$ at $T \gg eV$ and $\Delta\sigma_T \sim \ln V$ at $eV \gg T$ [the latter behavior follows from an estimate of the effective temperature of an electron which has undergone a tunneling: $T_e \sim eV$ in the layer $L_\varphi(T_e)/2$ near the tunneling gap]. This behavior of $\sigma_T(T, V)$ agrees qualitatively with the experimental data on Refs. 7 and 9. The details of the contribution of $\Delta\sigma_s$ to $\Delta\sigma_T$ depend strongly on the magnetic field and on spin scattering.

I wish to thank V. B. Sandomirskiĭ for discussion and support of this study and D. E. Khmel'nitskiĭ for useful discussion.

[1]L. P. Gor'kov, A. I. Larkin, and D. E. Khmel'nitskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **30**, 248 (1979) [JETP Lett. **30**, 228 (1979)].
[2]D. J. Thouless, Phys. Rev. Lett. **39**, 1137 (1977).
[3]B. L. Al'tshuler, D. E. Khmelnitzkiĭ, A. I. Larkin, and P. A. Lee, Phys. Rev. **B22**, 5142 (1980).
[4]B. L. Al'tshuler, A. G. Aronov, D. E. Khmel'nitskiĭ, and A. I. Larkin, Zh. Eksp. Teor. Fiz. **81**, 768 (1981) [Sov. Phys. JETP **54**, 411 (1981)].
[5]B. L. Al'tshuler and A. G. Aronov, Pis'ma Zh. Eksp. Teor. Fiz. **33**, 515 (1981) [JETP Lett. **33**, 499 (1981)].
[6]A. Kawabata, Solid State Commun. **34**, 431 (1980).
[7]J. M. Rowell, in: Tunneling Phenomena in Solids (ed. E. Burstein and S. Lundquist), Plenum, New York, 1969 (Russ. trans. Mir, Moscow, 1973).
[8]B. L. Al'tshuler and A. G. Aronov, Solid State Commun. **30**, 115 (1979).
[9]T. Carruthers, Phys. Rev. **B 10**, 3356 (1974).

Translated by Dave Parsons
Edited by S. J. Amoretty