# Texture effects in rotating superfluid $^3$He–B

G. E. Volovik
*L. D. Landau Institute of Theoretical Physics, Academy of Sciences of the USSR, Moscow*

A. D. Gongadze, G. E. Gurgenishvili, and G. A. Kharadze
*Institute of Physics of the Academy of Sciences of the Georgian SSR, Tbilisi*

M. M. Salomaa
*Low-Temperature Laboratory, Technical University, Helsinki*
*Project ROTA, Helsinki-Moscow-Tbilisi*

(Submitted 11 November 1982)
Pis'ma Zh. Eksp. Teor. Fiz. **36**, No. 11, 404–407 (5 December 1982)

In an inclined magnetic field a large shift of the NMR spectrum is expected to occur in the presence of rotation. For angles of inclination exceeding $\approx 14.5°$ a pair of linear topological defects, boojums, must appear on the cylinder wall. The texture in the vortexless state, which arises immediately after the onset of rotation, is investigated.

PACS numbers: 67.50.Fi

Recent NMR experiments in rotating superfluid $^3$He led to the discovery of quantized vortices in both superfluid phases.[1–3] In the *A* phase, a satellite peak[3] appears with rotation, which is related to the excitation of spin modes localized on nonsingular vortices.[4] In the *B* phase the vortices have a texture, which occurs in the cylinder in the presence of an axial magnetic field **H** and changes the spectrum of spin modes that are localized in the texture.[2] We shall examine a number of new texture effects, which should be observed in rotating $^3$He–B.

*1. Displacement of the NMR spectrum in an inclined field.* The order parameter in $^3$He–B, the orthogonal matrix $R_{ik}$ $(\mathbf{n},\theta_0)$, describes the rotation of the spin space relative to the orbital space by a large angle $\theta_0 = 104°$ around an arbitrary axis **n**. The angle $\beta$ between **n** and **H** determines the shift in the resonant frequency relative to the Larmor frequency $\omega_0$ in transverse NMR (see, for example, Ref. 5): $\omega-\omega_0=\sin^2\beta\Omega_L^2/2\omega_0$, where $\Omega_L \ll \omega_0$ is the Leggett frequency. In the absence of boundaries $\beta=0$ due to magnetic orientation energy we would have $F_M = -a(\mathbf{nH})^2$, and there is no shift. In a bounded geometry, a texture appears in the field **n** due to the orientational action of the surface [$F_S = -d(s_i R_{ik} H_k)^2$, where $s$ is the normal to the boundary]. Experimentally, the radius of the dish $R$ exceeds the characteristic magnetic length $\zeta_H$, so that the resonant frequency is shifted very slightly, and the texture leads only to the formation of equidistant satellite peaks corresponding to a discrete oscillator spectrum of spin waves which is localized in the texture.[5] The rotation of $^3$He–B in an axial field does not change the picture, leading only to an increase in the distance $\Delta\omega$ between the absorption peaks.[2]

The situation changes with rotation in an inclined field.[6] The orienting action of vortices on the vector **n**, which originates from the strong magnetic anisotropy inside the core of a vortex,[7] has the following form[6,7]:

$$F_V = \frac{2}{5} a\, \lambda (z_i R_{ik} H_k)^2, \tag{1}$$

where $\hat{\mathbf{z}}$ is the orientation of the axis of rotation. The parameter $\lambda$ is proportional to the vortex density and, therefore, to the angular rotational velocity $\Omega$. Minimizing $F_M+F_V$ shows that for nonvanishing angle $\mu$ between **H** and **z**, the angle $\beta$ differs from zero and is expressed in terms of $\lambda$ as follows:

$$u\cos 2\mu + \left(u^2-\frac{1}{2}\right)(1-u^2)^{-1/2}\sin 2\mu = \frac{1}{\lambda}, \quad u=1-\frac{5}{4}\sin^2\beta. \tag{2}$$

Equation (2) permits extracting the parameter $\lambda$ from the shift in the resonant frequency in an inclined field. Previously $\lambda$ was estimated in Ref. 7, starting from experimental data for the distance $\Delta\omega$ between peaks[2]: $\lambda \sim 2.8(0.9-T/T_c)$ or $0.6<T/T_c<0.8$. The new method is preferable, since it does not require a knowledge of the distribution $\mathbf{n}(\mathbf{r})$ in the texture, which is necessary for calculating $\Delta\omega$ theoretically; in addition, the shift in the frequency $\omega-\omega_0$ at $\mu=\pi/4$ must exceed $\Delta\omega$ by an order of magnitude.

2. *Texture in the intermediate vortex-free state.* After the $B$ phase begins to rotate in an axial field, a vortex-free state appears within a minute. In this state the normal component rotates as a rigid body $\mathbf{v}^n = [\vec{\Omega},\mathbf{r}]$, while the superfluid component is at rest $\mathbf{v}^s=0$.[2] The shape of the line in this state changes considerably due to the declination of **n** away from **H** at a large angle $\beta$ due to the orienting action of the counter flow:

$$F_f = 2/5\, a\, [(v_i^s - v_i^n) R_{ik} H_k]^2 / v_c^2,$$

where $v_c \sim 1$ mm/s. After the system of vortices is formed, the average $\langle \mathbf{v}^s\rangle = [\vec{\Omega},\mathbf{r}]$, and the initial spectrum reappears. The texture arising in the vortex-free state is determined by minimizing $F_M+F_f+F_g$, where the last term is the energy of deformation for the field **n** (Ref. 5):

$$F_g = 16/13\, a\, \xi_H^2 [(\nabla \mathbf{n})^2 - 1/2(\sqrt{5/8}\,\mathbf{n}\,\mathrm{rot}\,\mathbf{n} + \sqrt{3/8}\,\mathrm{div}\,\mathbf{n})^2] H^2. \tag{3}$$

In an axial field **n** is given by polar and azimuthal angles $\beta(r)$ and $\alpha(r)$:

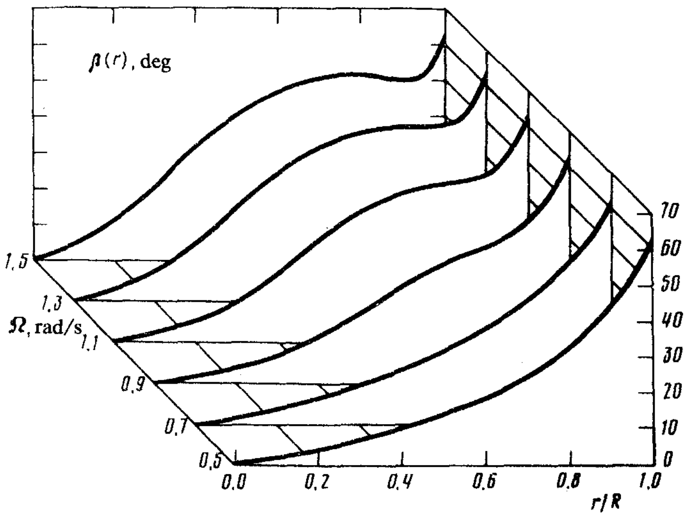


FIG. 1. Radial distribution of the angle $\beta$ between **n** and **H** in the texture arising in the metastable vortex-free state for different angular velocities of rotation of $^3$He–B at $T = 0.7\ T_c$ and $H = 284$ G.

$\mathbf{n} = \hat{\mathbf{z}} \cos\beta + (\hat{\phi} \sin\alpha - \hat{\mathbf{r}} \cos\alpha)\sin\beta$, where $\hat{\mathbf{z}}$, $\hat{\mathbf{r}}$, $\hat{\phi}$ are the unit vectors of a cylindrical system of coordinates. The surface energy $F_s$ determines the boundary conditions $\cos\beta(R) = \pm 1/\sqrt{5}, \tan\alpha(R) = \pm\sqrt{3}$. Figure 1 shows the result of minimization for $\beta(r)$ at $R = 2.5$ mm, $\xi_H = 12/H(\mathrm{G})$ cm$(H = 284$ G), which corresponds to a temperature $T/T_c = 0.7$ and for different values of $\Omega$.

If we are interested in the overall form $P(\omega)$ of the NMR spectrum, rather than in the position of the absorption peaks, then we can use the local oscillator approximation. Then $P(\omega) = 2/R^2 \int_0^R dr\, r\, \delta[\omega - \omega(r)]$, where $\omega(r) = \omega_0 + \sin^2\beta(r)\, \Omega_L^2/2\omega_0$ is the

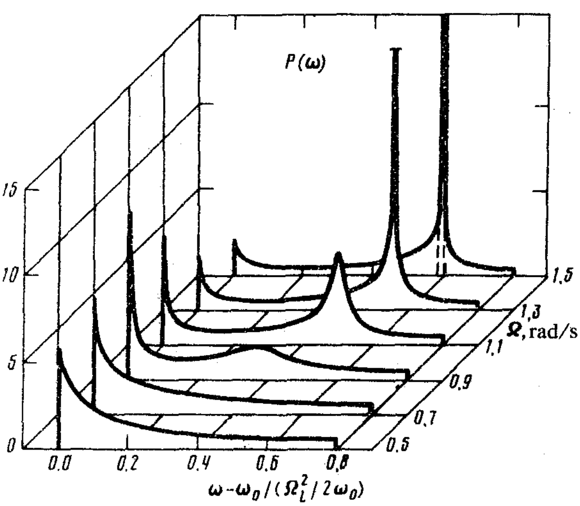


FIG. 2. Spectral density of absorption $P(\omega)$ as a function of the texture, shown in Fig. 1.

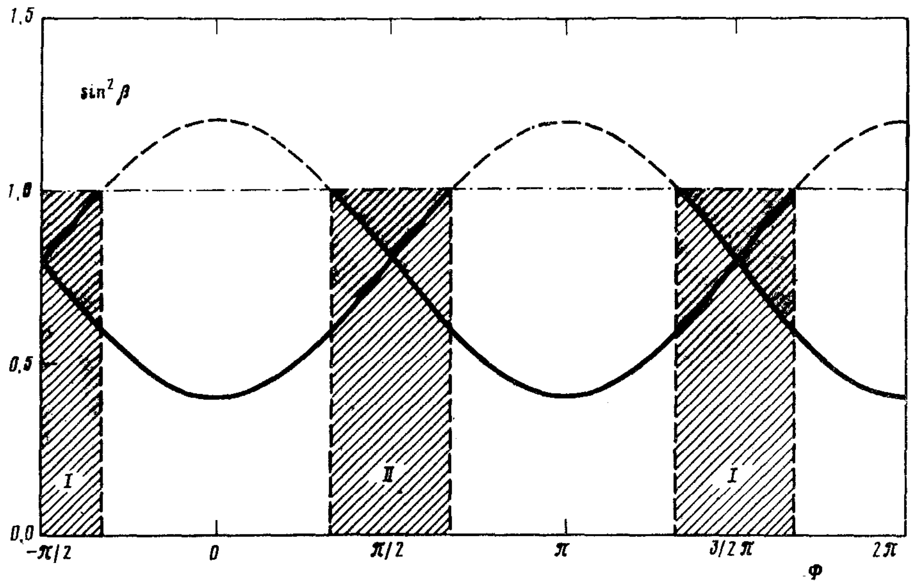


FIG. 3. Two branches of boundary conditions for $\sin^2\beta$ in a magnetic field oriented at an angle 30° to the axis of the cylinder. Boojums which go from one branch to another should occur in the shaded regions I and II.

local NMR frequency. The function $P(\omega)$ is shown in Fig. 2. For small $\Omega$ the spectrum is characterized by a sharp absorption peak near $\omega_0$. As $\Omega$ increases, the center of gravity of the spectrum is displaced toward higher frequencies (i.e., large $\beta$), and for $\Omega \sim 0.8$ rad/s a new peak appears. Then singularities, which originate from extrema of the function $\sin^2\beta(r)$, appear. The appearance of a new peak was observed experimentally[8] at $\Omega = 0.6$ rad/s. One of the possible reasons for the difference is the difference between $v_c$ and its value in the Ginzburg-Landau region.

*3. Linear boojums in an inclined field.* In an axial field there are four different orientations of the vector **n** on the boundary with the same energy $F_S$. For this reason, lines separating regions with different values of **n**, linear boojums, can appear on the surface. In an axial field these defects can exist only as metastable defects. In the case of an inclined field the dependence $\sin\beta$ at the boundary of the azimuthal angle $\phi$, measured from the plane $(\hat{\mathbf{z}},\mathbf{H})$, has two branches:

$$\sin\beta(R,\phi) = 2/\sqrt{5}\,(1 \pm \sin\mu\cos\phi)^{1/2} \tag{4}$$

with identical surface energy $F_S$. For $\sin\mu > 1/4$, each of the branches becomes discontinuous (see Fig. 3), which necessarily leads to the appearance of two boojums which separate the regions with different branches of $\sin\beta(R,\phi)$ on the surface. Boojums, for example, can occur on lines $\phi = \pi/2$ and $\phi = -\pi/2$. Since boojums affects the texture distribution of the vector **n** in the bulk, we can expect the spectrum to change for $\sin\mu > 1/4$.

We thank Yu. M. Bun'kov, O. T. Ikkala, M. Krusius, O. V. Lounasmaa, Yu. Simola, and P. Yu. Khakonen for useful discussions of the experiment.

[1]P. F. Hakonon, O. T. Ikkala, S. T. Islander, O. V. Lounasmaa, T. K. Markkula, P. Roubeau, K. M. Saloheimo, G. E. Volovik, E. L. Andronikashvili, D. I. Garibashvili, and J. S. Tsakadze, Phys. Rev. Lett. **48**, 1838 (1982).
[2]O. T. Ikkala, G. E. Volovik, P. Yu. Khakonen, Yu. M. Bun'kov, S. T. Islander, and G. A. Kharadze, Pis'ma Zh. Eksp. Teor. Fiz. **35**, 338 (1982) [JETP Lett. **35**, 416 (1982)].
[3]P. J. Hakonen, O. T. Ikkala, and S. T. Islander, Phys. Ref. Lett. **49**, 1258 (1982).
[4]H. K. Seppala and G. E. Volovik, submitted to J. Low Temp. Phys.
[5]D. D. Osheroff, Physica B **90**, 20 (1977).
[6]A. D. Gongadze, G. E. Gurgenishvili, and G. A. Kharadze, Fiz. Nizk. Temp. **7**, 821 (1981) [Sov. J. Low. Temp. Phys. **7**, 404 (1981)].
[7]P. J. Hakonon and G. E. Volovik, J. Phys. C. (in press).
[8]O. T. Ikkala, Ph.D. thesis.

Translated by M. E. Alferieff
Edited by S. J. Amoretty