Origin of the SU(5) symmetry of quarks and leptons

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Preons in a model with a unitary SU(3) metacolor form quarks and leptons in the ground state which show the standard SU(5) multiplets of symmetry $\bar{5} + 10$. The complete local symmetry of the quarks and leptons is $SU(6)_H \otimes SU(6)_V$ (a horizontal-vertical symmetry) and contains six SU(5) generations.

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An unbroken chiral symmetry quarantees massless fermions. If these fermions are preons which form observable quarks and leptons as a result of $SU(3)_{mc}$ metacolor forces, these quarks and leptons are also massless under the condition

$$\sum_q I_q A(q) = 3A(N), \tag{1}$$

according to 't Hooft.[1] Here $A(N)$ and $A(q)$ are the group coefficients of the triangle anomalies with respect to one of the groups, $SU(N)_L$ or $SU(N)_R$, of the chiral symmetry of the preons, $K(N) \equiv SU(N)_L \otimes SU(N)_R$ (N is the number of preons); $A(N)$ corresponds to a fundamental representation of N for preons, and $A(q)$ to a representation for massless composite fermions. The values of $|I_q|$ are the numbers of times the representation q appears in the spectrum of the composite fermions; for $I_q > 0$, these values correspond to left-hand chiral states, and at $I_q < 0$ they correspond to right-hand chiral states.

Condition (1) contains, through the coefficients $A(q)$, an explicit dependence on the number of preons, N. We adopt this condition, but instead of pursuing the 't Hooft approach[1] further and seeking N-independent solutions of Eq. (1) for the indices[1)] I_q, we will solve this equation for N, using in (1) only the ground states of bound fermions. For these states it is natural to allow only two possible values of $|I_q|$ in each representation q:

$$|I_q| = 0,1. \tag{2}$$

Let us assume that condition (1) has a solution for N at, say $N = N_0$. The chiral symmetry of the preons, $K(N_0)$, is thus also unbroken beyond their confinement radius, R_{mc}, and the composite fermions do not acquire masses on the order of their "sizes,"[1] $M \sim R_{mc}^{-1}$, in any representation q in (1). If this symmetry is a local symmetry, then it will obviously remain as a gauge symmetry of the composite quarks and leptons.

2. We introduce the N massless Dirac preons

$$\mathcal{P} \equiv (L, R); \quad L_{i\alpha}, \quad R_{ia} \qquad (i = 1, 2, 3), \tag{3}$$

where the index i corresponds to the metacolor with the local $SU(3)_{mc}$ symmetry which leads to preon confinement in the quarks and leptons (with a radius[3] $R_{mc} \lesssim 10^{-15}\ \mathrm{GeV}^{-1}$); $\alpha(1, \ldots, N)$ and $a(1,\ldots, N)$ are the indices of the symmetry groups $SU(N)_L$ and $SU(N)_R$, respectively, which are also gauge groups, by assumption.

Although the anomalies with respect to the $SU(3)_{mc}$ group have been cancelled, anomalies remain with respect to the $SU(N)_L$ and $SU(N)_R$ groups. To cancel them, we should[1] supplement states (3) with $SU(3)_{mc}$ metacolor singlet states of the opposite chirality (metacolor leptons or "leons," three right-handed and three left-handed):

$$r_\alpha^{(p)}, \ l_a^{(p)}; \quad p = 1, 2, 3. \tag{4}$$

We turn now from preons and leons to quarks and leptons. The currents of the composite fermions corresponding to ground bound states of three "valance" preons, colorless with respect to $SU(3)_{mc}$, are

$$\begin{aligned} \psi_{\alpha[ab]L}(x) &= L_{i\alpha}(R_{ja}\, C R_{kb})\, \epsilon^{ijk} \qquad && \boxed{L} \otimes \begin{array}{|c|}\hline R \\ \hline R \\ \hline\end{array}\,; \\ \chi_{\{\alpha[\beta\gamma]\}L}(x) &= O^A L_{i\alpha}(L_{j\beta}\, C\, O^A L_{k\gamma})\, \epsilon^{ijk}, \qquad && \begin{array}{|c|c|}\hline L & L \\ \hline L & \\ \hline\end{array} \\ O^A &= 1, \sigma_{\mu\nu}. \end{aligned} \tag{5}$$

The corresponding right-handed currents $\psi_R(x)$ and $\chi_R(x)$ are found from (5) through the interchange $L \leftrightarrow R$. The symmetry of the composite fermions with respect to the $SU(N)_L \otimes SU(N)_R$ group has thus been established [see the Young tableau in (5)], and we are in a position to solve systems (1) and (2). Fixing the values of the anomalous coefficients with respect to the $SU(N)_L$ group[1] in (1),

$$A(\psi_L) = \frac{1}{2}N(N-1), \quad A(\chi_L) = N^2 - 9, \quad A(\psi_R) = N(N-4), A(\chi_R) = 0, \tag{6}$$

and setting $A(N) = 1$, we find ($I_{\psi_L} = -I_{\psi_R} \equiv I_\psi, I_{\chi_L} \equiv I_\chi$)

$$-\frac{1}{2}N(N-7)I_{\psi}+(N^2-9)I_{\chi}=3. \tag{7}$$

It is easy to see that for the ground states [see (2)] Eq. (7) has the unique solution

$$I_{\psi}=1,\ I_{\chi}=0;\ N=6. \tag{8}$$

Beyond the radius R_{mc} we have thus obtained an anomaly-free $SU(6)_L \otimes SU(6)_R$ theory with multiplets of quarks and leptons, ψ_L and ψ_R, (5), and of leons, r and l, (4), which are massless in the symmetry limit (in the left-hand helicity basis $3\bar{r}+3l+\psi_L+\bar{\psi}_R$),

$$3\cdot(\bar{6},1)+3\cdot(1,6)+(6,15)+(\overline{15},\bar{6}). \tag{9}$$

It can be seen from (9) that if we identify $SU(6)_R$ with "vertical" symmetry which combines quarks and leptons within each generation, and if we identify $SU(6)_L$ with a "horizontal" symmetry, which transforms these generations between each other, then after a spontaneous breaking of this $SU(6)_H \otimes SU(6)_V$ to $SU(5)_V$ the multiplet (9) reduces rigorously to six standard $SU(5)_V$ generations because the heavy $(5+5)$ $SU(5)_V$ states of invariant mass drop out[4]:

$$\text{singlets}+3\cdot(5'+\bar{5})+6\cdot(5+\bar{5})+6\cdot(\bar{5}+10), \tag{10}$$

where we have singled out the 5′-plets which appeared from the leon fields (4).

3. By assumption, spontaneous symmetry breaking,

$$SU(6)_H\otimes SU(6)_V\to SU(5)_V\to SU(3)_c\otimes U(1)_{EM}, \tag{11}$$

is caused by composite Higgs scalars

$$(6,\bar{6}),\ (\bar{6},6),\ (35,1),\ (1,35),\ \ldots \tag{12}$$

with various preon-antipreon structures. These Higgs scalars arise along with quarks and leptons at distances $R \gg R_{mc}$.

It is clear, however, that these scalars cannot lead to the formation of the masses of the elementary leons.[4] For the leons to acquire masses, we need to introduce, in addition to the composite scalars in (12), the elementary Higgs scalars

$$(15,1),\ (1,\overline{15}). \tag{13}$$

The $SU(6)_H \otimes SU(6)_V$-symmetry Yukawa couplings of leons with composite fermions,

$$[(\bar{6},1)\ (6,15)]\cdot(1,\overline{15}),\qquad [(1,6)\ (\overline{15},\bar{6})]\cdot(15,1) \tag{14}$$

then give rise to leon masses which are proportional to the vacuum expectation values of fields (13). Under the assumption that these vacuum expectation values are comparable in magnitude, we find

$$\langle 15,1\rangle_0 \sim \langle 1,\overline{15}\rangle_0 \lesssim O(10^3)\,\mathrm{GeV}. \tag{15}$$

This result follows from the circumstance that the "vertical" scalar $(1,\overline{15})$ with an SU(5) content $(1,\bar{5}+\overline{10})$ breaks the vacuum expectation value up into an SU(5) $\bar{5}$-plet, which, on the other hand, causes a breaking of the ordinary $SU(2)\otimes U(1)$ symmetry.[4]

The leon 5′-plets which form "vector" combinations with composite $\bar{5}$-plets [see (10)], $3\times(5'+\bar{5})$, like the six generations of composite quarks and leptons,

$6\times(\bar{5}+10)$, should therefore have masses $\lesssim 0(1)$ TeV, since these masses and the masses of the weak $SU(2)\otimes U(1)$ bosons induce the same Higgs scalars. The other states in multiplet (10), singlets and composite 5-plets $6\times(5+\bar{5})$, acquire masses on the order of a combination $SU(6)_H \otimes SU(6)_V$ of the vacuum expectation values of composite scalars (12).

4. It has been suggested that the local preon symmetry $SU(N)_L \otimes SU(N)_R$ is also the horizontal-vertical symmetry of quarks and leptons, $SU(N)_H \otimes SU(N)_V$. It turns out that the only symmetry for which a spectrum of massless composite fermions can exist is $SU(6)_H \otimes SU(6)_V$.

The model predicts three new SU(5) quark–lepton generations, in good agreement with the description of the mass spectrum for known quarks and leptons.[5] In addition, this model predicts the existence of heavy leons: three weak lepton doublets of the type $(E^0/E)^{(p)}$ and three quark singlets $D^{(p)}$ with a charge of $-1/3$ ($p=1, 2, 3$).

The presence of new quark-lepton states in the mass interval 10–1000 GeV lets us raise the proton lifetime to 10^{31} yr or more, thereby overcoming the basic difficulty which the standard SU(5) model is currently having.[6] This question will be discussed in detail separately.

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[1]G. 't Hooft, Cargese Summer Institute Lectures, 1979.
[2]J. Preskill and S. Weinberg, Phys. Rev. **D 24**, 1059 (1981).
[3]A. A. Ansel'm, Pis'ma Zh. Eksp. Teor. Fiz. **31**, 88 (1980) [JETP Lett. **31**, 80 (1980)].
[4]L. B. Okun', Leptony i kvarki (Leptons and Quarks), Nauka, Moscow, 1981.
[5]Z. G. Berezhiani and Dzh. L. Chkareuli, Pis'ma Zh. Eksp. Teor. Fiz. **35**, 494 (1982) [JETP Lett. **35**, 612 (1982)].
[6]J. Ellis, CERN Preprint, Ref. TH-3174, 1981.

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