Stabilization of collisional modes for an explosive instability by drift effects

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The intrinsic modes for drift-collisional tearing (discontinuity) instability are studied in this work. It is shown that allowance for small corrections for the interaction of tearing perturbations with ion-acoustic waves leads to the disappearance of the proper solutions.

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It was shown[1,2] that the convective outflow of energy from the region of electron dissipation for tearing perturbations leads to a stabilization of the drift tearing modes (a similar effect is now known for drift waves[3–5]). It can be shown that the stabilization criteria obtained for the collisionless[1] and semicollisional[2] cases agree in accuracy and generally account for the interaction energy with ion-acoustic waves in the overall energy balance. The stabilization criterion for the collision mode of a drift tearing instability, obtained in Ref. 2 from the same considerations, is incorrect. As we

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shall show below, the intrinsic modes for this branch of oscillators are totally different from those found in Refs. 6 and 7 and used in Ref. 2 and, therefore, they vanish for small corrections for interaction with the acoustic ions.

We shall take into account the effect of collisions in a simple model of the Bhatnagar-Gross-Crook collisional term. The system of equations for the perturbations of the scalar ϕ and vector A potentials has the form

$$\eta_i \frac{\rho_i^2}{2}\phi''_{xx} = \left(\phi - \frac{\omega A_{\|}}{k_{\|} c}\right)\left\{\frac{k_{\|}^2 v_{Te}^2}{k_{\|}^2 v_{Te}^2 - 2 i \omega \nu_{ei}}\right\}\left[\overline{\delta\omega_e} - \eta_i \frac{k_{\|}^2 v_{Ti}^2}{2\omega^2}\right] = S, \quad (1)$$

$$\frac{\omega - \omega_*^e}{\omega} = \delta\omega_e, \qquad \frac{\omega - \omega_*^i}{\omega} \approx \eta_i = \frac{T_e + T_i}{T_i}; \quad k_{\|}(x) = \frac{\mathbf{k}\mathbf{B}(x)}{\mathbf{B}}, \quad (2)$$

$$A''_{xx} = \frac{2\omega_{pe}^2}{v_{Te}^2}\,\frac{\omega S}{k_{\|} c}.$$

Here $\omega_* = \omega_*^e$ is the electron drift frequency, ν_{ei} is the electron-ion collision frequency, $\omega_*^i \approx -\omega_* T_e/T_i$, $\nu_{ei} \gg \omega \gg \nu_{ie} = \nu_{ei}\, m/M$, and $\omega \sim \omega_* \gg \mathrm{Im}\,\omega$. The singular surface $k_{\|} = 0$ coincides with the plane $x = 0$, and the expansion for the ion contribution in Eq. (1) is valid to distances $|x| < \delta_i = \omega_*/k'_{\|} v_{Ti}$ and $k'_{\|} = \partial k_{\|}(x)/\partial x = k/L_s$, where L_s is the characteristic shear length.

The basic exchange of perturbation energy with the electrons occurs inside a resistive region $|x| < \delta_e = \sqrt{2\omega_* \nu_{ei}/k'_{\|} v_{Te}}$, and the total free energy of the instability stored in the global magnetic field configuration is proportional to the known parameter Δ' in the tearing mode theory, which formally corresponds to the discontinuity in the logarithmic derivative of the external solution on the boundary of the inner region. Moreover, the dispersion equation can be obtained by integrating the equation for A_{xx} over the entire inner region [assuming that $A(x) \sim \mathrm{const}$] and by connecting the logarithmic derivative for the interior and exterior solutions (see, e.g., Refs. 8 and 9)

$$\Delta' = (A(0))^{-1} \int_{-R}^{+R} \frac{2\omega_{pe}^2}{v_{Te}^2}\,\frac{\omega}{k_{\|} c}\, S\, dx = \Delta_e + \Delta_i^{(1)} + \Delta_i^{(2)}, \quad (3)$$

$$R \to \infty$$

the Δ_e term, which was taken into account earlier,[6,7] arises in the integration of the electron contribution in Eq. (2) with the potential $\phi^{(0)}$, which is determined by the electron term in Eq. (1). The ion contribution in Eqs. (1) and (2), i.e., the interaction between the tearing mode and ion-acoustic oscillations, is taken into account as a perturbation. This leads to the appearance of a correction $\phi^{(1)}$ to the potential $\phi^{(0)}$ and to two corresponding ion terms in Eq. (3)

$$\Delta_e = \frac{\omega_{pe}^2}{c^2}\,\frac{\omega_*}{\nu_{ei}}\,(-i\delta\overline{\omega_e})\int_{-\infty}^{+\infty}(1+y_0x)\,dx, \tag{4}$$

$$\Delta_i^{(1)} = \frac{\omega_{pe}^2}{c^2}\,\frac{\omega_*}{\nu_{ei}}\,\frac{i\eta_i}{2}\,\frac{1}{\delta_i^2}\int_{-\infty}^{+\infty}x^2(1+y_0x)\,dx, \tag{5}$$

$$\Delta_i^{(2)} = \frac{\omega_{pe}^2}{c^2}\,\frac{\omega_*}{\nu_{ei}}\,\frac{i\eta_i}{2}\,\frac{1}{\delta_i^2\,\delta^4}\int_{-\infty}^{+\infty}xy_1(x)\,dx, \tag{6}$$

$$y_0 = -\frac{k_{\parallel}'\,c\,\phi^{(0)}}{\omega_*\,A_{\parallel}}, \quad y_1 = \frac{2k_{\parallel}'c\,\delta_e^2\rho_i^2\delta_i^2}{i\omega A_{\parallel}}\,\phi^{(1)}, \quad \delta^4 = \frac{\delta_e^2\rho_i^2\eta_i}{i\delta\overline{\omega_e}},$$

$$\eta_i = 1 + \frac{T_e}{T_i}.$$

The y_0 and y_1 potentials satisfy the following equations:

$$y_0'' - y_0x^2/\delta^4 = x\delta^{-4}, \tag{7}$$

$$y_1'' - y_1x^2\delta^{-4} = x^3(1+y_0x). \tag{8}$$

The parameter δ determines the characteristic length of variation of the longitudinal electric field[8] and, in the case of the collisional mode, it is small in comparison with δ_e.

The solution of Eq. (7) can be written as a sum of the special and general solutions

$$y_0(x/\delta) = \frac{-x}{2\delta^2}\int_0^{\pi/2}d\theta\sqrt{\sin\theta}\,e^{\frac{-x^2\cos\theta}{2\delta^2}} + C_1\sqrt{\frac{x}{\delta}}\,I_{1/4}\left(\frac{x^2}{2\delta^2}\right) + C_2\sqrt{\frac{x}{\delta}}\,I_{-1/4}\left(\frac{x^2}{2\delta^2}\right), \tag{9}$$

where $I_{\pm 1/4}(\ \ )$ is a Bessel function of imaginary argument. Since the desired solution $\phi^{(0)}$ should be odd, $C_2 \equiv 0$, and C_1 can be determined from the condition for the finiteness of the solution as $|x| \to \infty$. If $\mathrm{Re}\,\delta^2 > 0$, $C_1 = 0$; and if $\mathrm{Re}\,\delta^2 < 0$, $C_1 = \sqrt{2\pi}\Gamma(\frac{3}{4})\delta^{-1}$, where $\Gamma(\frac{3}{4})$ is the gamma function. Thus, if $\delta = |\delta|e^{i\alpha}$, then the solution changes abruptly on the lines $\alpha = \pm\pi/4$ (Stokes lines).

We shall first examine the problem that was solved in Refs. 6 and 7, i.e., we omit the ion contributions in Eqs. (1) and (2), we retain only the first term in Eq. (3). By extending the solution analytically from the real axis to the straight line $t\exp(i\alpha)$, ($\mathrm{Re}\,\delta^2 > 0$) or to the straight line $t\exp i(\alpha - \pi/2)$, ($\mathrm{Re}\,\delta^2 < 0$), where $t \in (-\infty, +\infty)$ is a real number, we can see that the integral $I=2\int_0^\infty dx(1+y_0x)$ has the form

$$I=\begin{cases}(-1)^n I_1\delta, & \left(\frac{-\pi}{4}+\pi n<a<\frac{\pi}{4}+\pi n\right) \quad \begin{matrix} n=0, & \text{I sector} \\ n=1, & \text{III sector}\end{matrix} \\ \\ -i(-1)^k I_1\delta & \left(\frac{\pi}{4}+\pi k<a<\frac{3\pi}{4}+\pi k\right) \quad \begin{matrix} k=0, & \text{II sector} \\ k=1, & \text{IV sector},\end{matrix}\end{cases} \tag{10}$$

where $I_1=2\pi\Gamma(3/4)/\Gamma(1/4)$.
Assuming that the solution δ lies in the first sector, we obtain the dispersion equation which fully agrees with the equations in Refs. 6 and 7

$$\frac{c^2}{\omega_{pe}^2}\Delta'=\frac{\omega_*}{\nu_{ei}}(-i\overline{\delta\omega_e})\,I_1\delta\,,\qquad \delta^4=\frac{\delta_e^2\rho_i^2\eta_i}{i\overline{\delta\omega_e}}\,. \tag{11}$$

Solving Eq. (11), we obtain a contradiction with the initial assumptions not noted in Refs. 6 and 7: $\delta\sim\sqrt[3]{-1}$, i.e., all three roots lie outside of the sector I. Assuming similarly that the solution lies in the sectors II, III, and IV, respectively, and selecting the required phase according to Eq. (10), we arrive each time at a contradiction with the initial assumptions. Thus, *there are no solutions lying outside the Stokes lines* $a=\pm\pi/4$.

Let us now assume that the solution δ has a phase exactly equal to $\pi/4$. Thus, the solution on the Stokes line must be constructed as the half-sum of the solutions from above and from below, i.e., $C_1=\sqrt{\pi/2}\,\Gamma(\frac{3}{4})\delta^{-1}$ (only this constant, as we shall see below, guarantees the existence of intrinsic modes). Moreover, this solution coincides with the solution selected from the condition for the convective flow of energy in Ref. 1. Thus, instead of Eq. (11) we have

$$\frac{c^2}{\omega_{pe}^2}\Delta'=\frac{\omega_*}{\nu_{ei}}(-i\overline{\delta\omega_e})\,I_1\frac{\delta-i\delta}{2}\,. \tag{12}$$

It can be easily seen that Eq. (12) has a root $\delta\sim\sqrt[3]{i-1}$ with the phase $\pi/4$, which corresponds to the increasing solution $\mathrm{Re}\overline{\delta\omega_e}=0$, $\mathrm{Im}\overline{\delta\omega_e}>0$. The increment in fact coincides with that in Refs. 6 and 7, although the solution has a completely different nature.

We shall now solve the final problem: whether such unusual solution is stable relative to the corrections $\Delta_i^{(1,2)}$. If allowance for Δ_i changes the phase and the solution leaves the Stokes line, then the intrinsic oscillation modes, as we have seen above, are absent and there is no instability. The perturbed solution y, on the Stokes line is constructed similarly to Eq. (12), as the half sum of the solutions from above and from below. This makes it possible to determine the phase $\Delta_i^{(2)}$. As for the $\Delta_i^{(1)}$, we can show [after calculating the integral (5)] that, in the case of solution (9), it turns out to be identically equal to zero

$$\Delta_i^{(1)} = 0, \quad \Delta_i^{(2)} = \frac{\omega_{pi}^2}{c^2} \frac{\omega_*}{\nu_{ei}} \frac{i\eta_i}{2} \frac{\delta^3}{\delta_i^2} I_2 \frac{1+i}{2}, \tag{13}$$

where $I_2 = \int_{-\infty}^{+\infty} t\bar{y}_1(t)dt \sim -3$ is the integral along the real axis from the $\bar{y}_1(t)$ function that satisfies the equation

$$\bar{y}_1'' - t^2\bar{y}_1 = t^3\left(1 - t^2/2\int_0^{\pi/2} d\theta\sqrt{\sin\theta}\,\exp\left(-\frac{t^2\cos\theta}{2}\right)\right). \tag{14}$$

It is easy to see that the phase of Eq. (13) on the Stokes line $\delta \sim e^{i\pi/4}$ is equal to $\pi/2$, i.e., allowance for the ion correction shifts the solution from the Stokes line (allowance for the corrections due to the finiteness of δ/δ_e gives a similar result). Thus, although the solution exists formally, it vanishes when small corrections are taken into account. There are no intrinsic modes for the collisional tearing instability for drift frequencies which are comparable to the well-known Furth *et al.*[8] increment: $\omega_* > \nu_{ei}^{3/5}(k'_{\parallel} v_{Te}/\rho_i)^{2/5}(c^2\Delta'/\omega_{pe}^2)^{4/5}$.[1)] The stabilization criterion[2] for the examined mode can be determined from our Eqs. (3)–(6), if it is assumed regardless of the phase relations, that the ion correction $\Delta_i^{(2)}$ attains a finite value of the order of Δ'. However, as we showed above, the solution is such that the already small interaction with the ions leads to the disappearance of the intrinsic solutions for the drift collisional oscillation mode.

[1)]However, for very strong collisions this criterion is replaced by a weaker constraint $\omega_* > \nu_{ei}(m/M)$, since for $\nu_{ei} > \omega_*(m/M)$ the drift effects no longer play a role.

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