Kinetics of parametric instability of elastic waves in a dielectric

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Parametric excitation of acoustic waves in a dielectric due to elastic nonlinearity of the medium is investigated. It was found that in the initial stage of excitation the phonon density increases exponentially with an instability increment which coincides with a known value from the parametric excitation theory.

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The parametric excitation of transverse elastic waves with frequencies 3/7 and 4/7 of the frequency of a longitudinal elastic pump wave in a paratellurite (TeO_2) single crystal was initially described in Ref. 1.

A self-modulation effect of parametrically excited waves has also been observed.[2] Such parametric excitation can be represented as an interaction of two transverse phonons with oppositely directed wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ and a longitudinal phonon with the wave vector $\mathbf{k}_3$. The laws of conservation of energy and momentum in this case are satisfied: $\omega_3 = \omega_1 + \omega_2$ and $\mathbf{k}_3 = \mathbf{k}_1 + \mathbf{k}_2$. Thus, the parametric generation—decay of a longitudinal phonon with the frequency ω_3 into two transverse phonons with frequencies ω_1 and ω_2 occurs when the pumping power P_3 exceeds the threshold value which is determined by the expression

$$P_{3\,\text{thresh}} = 8\,\frac{\rho v_L^3 S}{\Gamma^2 \omega_1 \omega_2 \tau_1 \tau_2}, \tag{1}$$

where ρ is the density of the medium, v_L is the velocity of the longitudinal wave, Γ is the nonlinear parameter, τ_1 and τ_2 are the relaxation times for the transverse waves, and S is the cross section of the acoustic beam.

The kinetics of the parametric instability of elastic waves in a dielectric heretofore have not been investigated.

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The paratellurite single crystal has a strong acoustic anisotropy in the [110] direction. The propagation velocity of the longitudinal wave in this direction is $v_L = 4.21\times10^5$ cm/sec, and that of the transverse wave is $v_T = 0.616\times10^5$ cm/sec, and the damping of transverse waves, which greatly exceeds that of the longitudinal waves, reaches 290 dB/cm·GHz^2. As a result of propagation of the elastic waves in paratellurite along this axis, we can observe a pure three-frequency interaction at which almost all the pump energy is converted to parametrically excited waves, and the excitation of higher pump harmonics can be ignored.[2]

Experimental studies were performed using a paratellurite sample with dimensions $0.5\times0.5\times1$ cm^3. An elastic wave piezoemitter was placed at one end of the crystal and an absorber was placed at the other. The piezoemitter was excited at a frequency of 0.5 GHz (pump frequency) and the frequencies of the parametrically excited waves were 0.285 and 0.215 GHz. The elastic waves in the sample were studied by means of the Bragg acoustical-optical diffraction. Diffraction occurs as the laser beam strikes the sample ($\lambda = 0.63\ \mu$), in which the elastic wave is propagated (wavelength Λ) at an angle θ to this wave, which satisfies the equation $\sin\theta = \lambda/2\Lambda$. We can determine from the diffraction angle the type and frequency of the elastic wave. Since the relative diffraction intensity η for small efficiencies is proportional to the power of the elastic wave $\eta \approx \frac{\pi^2 n^6 p^2 l^2}{2\lambda^2 \rho v^3 S} P$ (where n, p and l are the index of refraction, photoelastic constant, and the length of the piezoemitter, respectively), the acoustic power at a given point in the sample can be determined from the diffraction intensity.

When the pumping power P_3 is greater than the threshold power $P_{3\text{thr}}$, a shearing of the pump pulse at time t_0 was observed, and at the same time the amplitude of the parametrically excited waves sharply increased. The shear depth is a function of the crystal coordinate and of the pump power.[2] The time from the onset of the pulse to shear t_0 is related to the quantity $(\sqrt{P_3/P_{3\text{thr}}} - 1)^{-1}$ by a linear dependence. The time τ (from the onset of the shear to the steady-state conditions for any, even the smallest, above-threshold pumping is much less than t_0 (5–50 μsec).

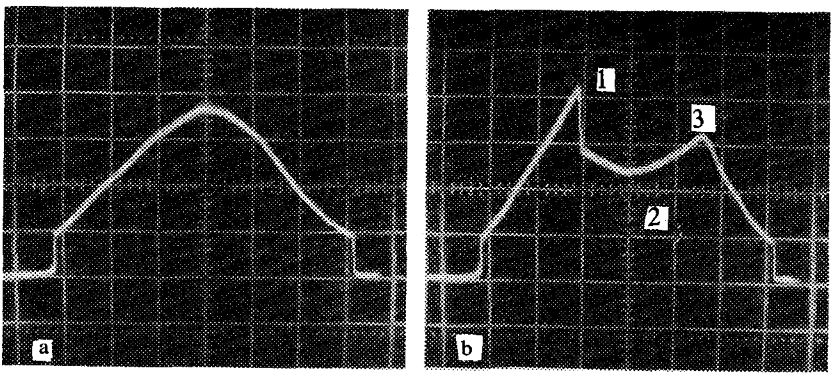

FIG. 1. Oscillograms of pump pulses: (a) power below threshold; (b) power above threshold; 1—onset of generation; 2—steady-state regime; 3—disruption of generation. Scanning rate is 2 μsec/division.

To determine the nature of the onset, development, and disappearance of the parametric generation, we used triangular pump pulses, in contrast to the methodology proposed in Ref. 3. The experiments with triangular shaped pulses enabled us to determine the pump power for the onset and disappearance of generation, the pump power in the steady-state regime, and the onset and disruption of generation (Fig. 1). Evidently, the power at which the generation occurs depends on the rate of increase of the pump power. In the cw mode, the power at which the generation occurs and is disrupted coincides with the threshold power.

In the steady-state, the pump power decreases below the threshold. This is attributed to the shape of the pump pulse. The pumping in the steady-state parametric regime has a nonuniform distribution along the length of the crystal,[2] and the oscillograms are taken at the point where the pump level decreases almost to zero.

The law for the increase of the number of phonons in the initial stage of the process (up to time t_0) was studied by stopping the pumping for a time δ.[4] We obtained the dependence of the onset of shear t_2 on δ and t_1 (Fig. 2). The data indicate that at $t_2 < t_0$, t_2, t_1, and δ are related by the relation $t_2 = t_0 - t_1 + \delta(\sqrt{P_3/P_{3\text{thresh}}} - 1)^{-1}$. This means that the number of parametrically excited phonons increases according to the exponential law:

$$n_{1,2} = n_{T1,2} \exp\{(\sqrt{P_3/P_{3\text{thres}}} - 1)t/\tau_{1,2}\}, \tag{2}$$

where $n_{T1,2}$ is the thermal density of the transverse phonons. The instability increment $\xi_{1,2} = (1/\tau_{1,2})(\sqrt{P_3/P_{3\text{thr}}} - 1)$ coincides with a known increment from the theory of parametric excitation.

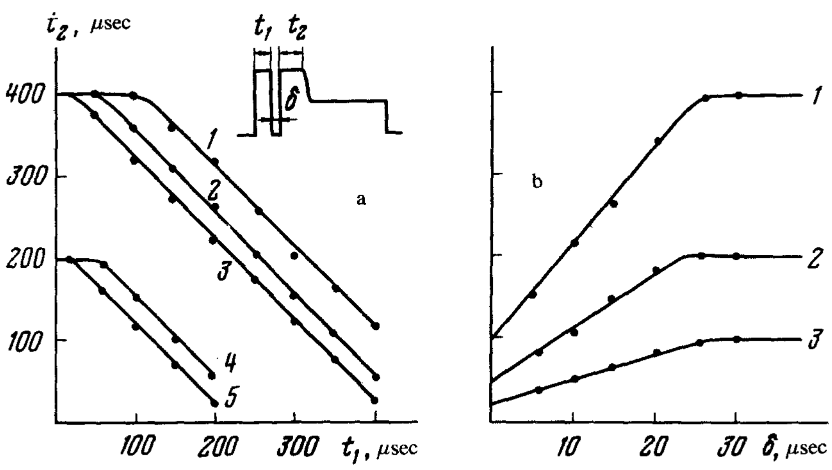

FIG. 2. Dependences of shear onset t_2: (a) from the time of disruption t_1: 1—$P_3/P_{3\text{thr}} = 1.18$, $\delta = 10$ μsec; 2—$\delta = 5$ μsec; 3—$\delta = 2$ μsec; 4—$P_3/P_{3\text{thr}} = 1.38$, $\delta = 5$ μsec; 5—$\delta = 2$ μsec; 6—from time δ: 1—$P_3/P_{3\text{thr}} = 1.18$, $t_0 = 400$ μsec, $t_1 = 300$ μsec; 2—$P_3/P_{3\text{thr}} = 1.38$, $t_0 = 200$ μsec, $t_1 = 150$ μsec; 3—$P_3/P_{3\text{thr}} = 1.84$, $t_0 = 100$ μsec, $t_1 = 80$ μsec. The inset shows the pump pulse with a disruption for a time δ.

The conducted experiments allow us to draw the following conclusions. For the case of pulsed pumping there exists a substantial power difference between the onset and collapse of the generation which increases with increasing rate of the pump power. In the cw regime, there is no such difference. This leads us to assume that the excitation regime is "soft." The characteristic shape of the pump pulse (with the shear) can also be observed, as indicated in Ref. 4, in the "soft" mode. In the case of parametric excitation of magnons in antiferromagnetic materials,[3] the excitation regime is "hard," and the instability increment differs from that predicted by theory. In all probability, this difference in the behavior of magnons and phonons is related to the fact that for parametric excitation of phonons in dielectrics there is no negative nonlinear attenuation.

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