Conductivity of a two-dimensional electron gas

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A correction for the free-path time in a random-impurity field and the corresponding correction for the conductivity are calculated for a two-dimensional, degenerate electron gas with a Coulomb interaction.

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In recent studies of the problem of localization of two-dimensional electrons, several mechanisms have been considered,[1,2] which decreased the conductivity $\sigma(T)$ as a result of decreasing the temperature. In Ref. 1 the negative corrections for σ took into account the interference due to scattering of noninteracting electrons by the different impurities. In Ref. 2, as applied to a three-dimensional case, such a correction was due to interference of the impurity and electron-electron interaction. Extrapolation of the Ref. 2 method to the two-dimensional case with a short-lived, interparticle potential gives a correction $\delta\sigma/\sigma_0 \sim (\epsilon_F \tau_0)^{-1} \ln T\tau_0$, where ϵ_F is the Fermi energy, τ_0 is

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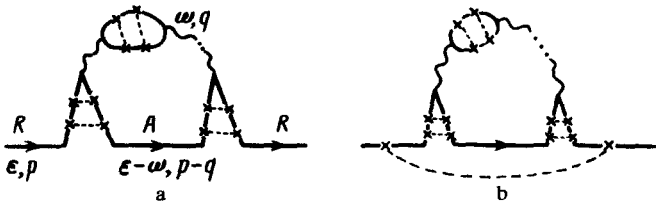

FIG. 1.

the transit time disregarding the electron-electron interaction. The same result, within the accuracy of the numerical factor, follows from Ref. 1. The logarithmic dependence of σ on T was observed recently in thin (~ 3 nm) films of AuPd.[3]

In this article we show that under the conditions where the interparticle potential is considered a two-dimensional Coulomb potential, the interference of the impurity and interparticle scattering decreases the conductivity more rapidly $\delta\sigma/\sigma_0 \sim -(\epsilon_F\tau_0)^{-1} \ln^2 T\tau_0$. Such behavior is expected in the semi-metal films. For example, in the case of Bi, where the Debye radius (three-dimensional) is $\kappa_D^{-1} = 100$ Å,[4] a film of several nanometer thickness satisfies the postulated conditions.

The first correction, with respect to the small parameter $(\epsilon_F\tau_0)^{-1}$, to the self energy Σ of the electron is shown in Fig. 1. The standard "cross" technique for the impurity averaging is used; the wavy line represents the interparticle Coulomb potential. The special term Σ^s is obtained from the integration region $\omega\tau_0 < 1$, $ql < 1$. Since all the internal frequencies and energies are greater than the temperature, we can go over to the limit $T \to 0$ in the expression for Σ^s. Thus, we obtain the following expression for the diagram in Fig. 1a:

$$a\Sigma^s_{\epsilon>0}(p) \approx \int_{\epsilon}^{\tau_0^{-1}} \frac{id\omega}{2\pi} \int \frac{qdq}{2\pi} \left[\frac{2}{(ql)^2 - 2i\omega\tau_0}\right]^2 \frac{2\pi e^2}{\epsilon_\infty q}\left(1 + \frac{l^2\kappa q}{(ql)^2 - 2i\omega\tau_0}\right)^{-1} \times (\epsilon - \xi_p - i/2\tau_0)^{-1}, \quad (1)$$

where ϵ is the electron energy measured from the chemical potential, $l = v_F\tau_0$, and $\kappa = 2e^2m/\epsilon_\infty$. The first factor in Eq. (1) is due to interference of the impurity and interparticle interactions and the second one is a screened, two-dimensional Coulomb potential, where the following expression is substantial for the polarization loop[5]

$$-(m/\pi)(ql)^2/[(ql)^2 - 2i\omega\tau_0)]. \quad (2)$$

The last factor is due to $G^A_{\epsilon-\omega}(p-q)$, where, as we shall see from the end result, the dependence on ω and q can be disregarded in the logarithmic approximation. The ω is bounded below due to the requirement that the pole of the Green's energy function of the particle should go over to the other side of the real axis after emitting the interaction quantum. Otherwise, the impurity corrections for the Coulomb potential are small.

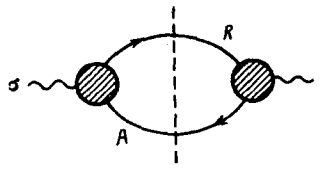

FIG. 2.

Integrating with respect to the frequency ω, we obtain

$$^{a}\Sigma^{s} \approx -\frac{i}{\tau_0}(2/\pi\epsilon_F\tau_0)\ [1+2i(\epsilon-\xi_p)\tau_0]^{-1}\int_0^1 \frac{dy}{y}\ln\frac{2\epsilon\tau_0+i\kappa l y}{2\epsilon\tau_0+iy^2}. \tag{3}$$

It can be seen that integration region with respect to $y=ql$, which account for the major contribution to the integral, is $\epsilon\tau_0/\kappa l<y<1$. Calculation of this integral and a similar expression corresponding to Fig. 1b gives the formula:

$$G_\epsilon^R(p)^{-1} = \epsilon-\xi_p+i/2\tau_0 - i(16\pi\epsilon_F\tau_0^2)^{-1}[(\epsilon-\xi_p+i/2\tau_0)/(\epsilon-\xi_p-i/2\tau_0)] \times \ln^2\epsilon\tau_0 \tag{4}$$

Since σ is proportional to the imaginary part of the current correlator, for which a unitarity condition exists,[6] the following formula holds for isotropic scattering by impurities (Fig. 2).

$$\sigma \sim \int d^2p\, d\epsilon\, (-\partial n_F/\partial\epsilon)\, A^2(\epsilon, p)\,|\vec{\Gamma}(\epsilon, p)|^2, \tag{5}$$

where $A(\epsilon, p)$ is a step of function $G_\epsilon(p)$ on the cut $\mathrm{Im}\epsilon=0$ and Γ is the vector vertex. The main contribution to the integral (5) comes from the region $\epsilon-\xi_p\sim\tau_0^{-1}$, $\epsilon\sim T$. In zeroth approximation $\Gamma_0\sim\mathbf{p}$, we obtain

$$\sigma(T)/\sigma_0 = 1-(4\pi\epsilon_F\tau_0)^{-1}\ln^2 T\tau_0. \tag{6}$$

The vector vertex in the first approximation is given by the expression in Fig. 3. Here both internal electron Green's functions in the correction for Γ_0 are simultaneously either leading or lagging; hence, the impurity inclusions are small. In addition, the momentum integral in Γ_1 is collected from a narrow region $\epsilon'-\xi_{p-q}\sim\tau_0^{-1}$, and the frequency integral is corrected from a wide region $\epsilon_F<\epsilon'<0$; therefore, there are no corrections that are logarithmically dependent on T. Thus, allowance for Γ_1 reduces to an ordinary renormalization of electron velocities at the Fermi surface.

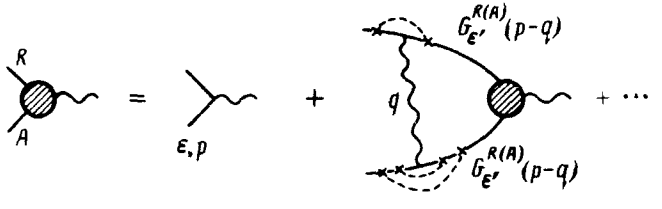

FIG. 3.

To use Eq. (6), we must satisfy the condition $\ln(T\tau_0)^{-1} > 1$, in addition to the aforementioned conditions. We note that $\kappa l = (e^2/\epsilon_\infty v_F)\epsilon_F \tau_0$ can be used for Bi of the order of $0.1\epsilon_F\tau_0 \gtrsim 1$ for slightly contaminated samples.

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