# Solutions of the classical Yang-Mills equations containing instantons and merons

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A class of solutions of the Yang-Mills equations in the Euclidean space was found and analyzed without imposing boundary conditions on the potentials. The solutions contain instantons and merons on an equal basis.

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Recently, an interest in the classical solutions of the Yang-Mills equations without external sources has increased.[1-5] This interest is largely connected with the hope of understanding the problems of quark confinement and the structure of the vacuum in non-Abelian gauge theories.

In the Euclidean space, the solutions in which the action functional has finite values (instantons) are remarkable.[1] Recently, meron solutions have been found.[2]

The most interesting solutions in the Minkovski space are those with a finite energy.[3]

In this paper we investigate a new class of solutions of the Yang-Mills equations without sources in the Euclidean space and without imposing boundary conditions on the potentials.

We seek a solution of the Yang-Mills equations for the SU(2) group

$$\nabla_\mu^{ab} G_{\mu\nu}^b(x) = 0 \tag{1}$$

($\nabla_\mu^{ab}$ is the covariant derivative, $G_{\mu\nu}^a$ is the field tensor, $a = 1, 2, 3$, and $\mu = 0, 1, 2, 3$) in the Euclidean space in the form

$$A_\mu^a(x) = \eta_{\mu\nu}^a x_\nu f(\xi), \tag{2}$$

where $\eta_{\mu,\nu}^a$ is the known t'Hooft tensor[5] and $\xi = (x_\mu x_\mu)^{1/2}$. The equation for the $f(\xi)$ function follows from Eqs. (1) and (2):

$$f''(\xi) + \frac{5}{\xi} f'(\xi) + 6gf^2(\xi) - 2g^2\xi^2 f^3(\xi) = 0\,, \tag{3}$$

which has an exact solution, so that for the potential we have

$$A_\mu^a(x) = \frac{\eta_{\mu\nu}^a x_\nu}{g\xi^2}\left\{1 + \left(\frac{2k^2}{1+k^2}\right)^{1/2} \sin\left[\left(\frac{2}{1+k^2}\right)^{1/2} \ln\left(\frac{\xi}{\xi_0}\right);\ k\right]\right\}\,, \tag{4}$$

where $\xi_0$ and $k$ are integration constants and $\sin[x; k]$ is the elliptic Jacobi sine of the modulus $k$ ($0 \leqslant k \leqslant 1$). Introducing the notation $A_\mu \equiv A_\mu^a \tau^a$, where $\tau^a$ are generators of the SU(2) group, we can write the solution of Eq. (4) in terms of the $g(x)$ matrices of the SU(2) group[1]:

$$A_\mu(x) = \frac{1}{2}\left\{1 + \left(\frac{2k^2}{1+k^2}\right)^{1/2} \operatorname{sin}\left[\left(\frac{2}{1+k^2}\right)^{1/2} \ln\left(\frac{\xi}{\xi_0}\right);\ k\right]\right\} g^{-1}(x)\partial_\mu g(x). \tag{5}$$

As can easily be seen, at $k = 0$ and 1, Eq. (5) acquires a single-meron and a single-instanton configuration, respectively. For the remaining values of $k$, the boundary conditions cannot be imposed on $A_\mu(x)$ when $\xi \to \infty$, because of the second term in Eq. (5); as a result, the action functional in the solutions of Eq. (5) is undefined for any $k \neq 0.1$. This explains the continuous transition from an instanton to a meron which has different topological charges[1,2] in the obtained solution of Eq. (5).

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[1]A.A. Belavin, A.M. Polyakov, A.S. Schwartz, and Yu.S. Tyupkin, Phys. Lett. **59B**, 85 (1975).
[2]V. de Alfaro, S. Fubini, and G. Furlan, Phys. Lett. **65B**, 163 (1976); C. Calan, R. Dashen, and D. Gross, Phys. Rev. **17D**, 2717 (1978).
[3]M. Luscher, Phys. Lett. **70B**, 321 (1977).
[4]G.Z. Baseyan, S.G. Matinyan, and G.K. Savvidi, Pis'ma Zh. Eksp. Teor. Fiz. **29**, 641 (1979) [JETP Lett. **29**, 587 (1979)].
[5]G. t'Hooft, Phys. Rev. Lett. **37**, 8 (1976).