Diffusion in toroidal traps with allowance for an electronic anomaly

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It is shown that plasma diffusion observed in tokamaks can be explained by anomalous electron viscosity. The diffusion flow in this case may vary only within the limits of neoclassical and pseudoclassical flows. The remaining coefficients are related to the diffusion coefficient by specific relations, accounting of which in the numerical models gives a more complete description of the transport processes.

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Analysis of the transport effects in a tokamak on the basis of neoclassical theory[1] produces results that differ from the experimental results in many parameters. Thus, for example, the confinement times of the plasma and of the electron energy are 10 to 100 fold smaller than those predicted theoretically. Therefore, the empirical coefficients, which exceed considerably the neoclassical coefficients, must be used in modeling the transport processes. Given that the confinement times are close to the experimental values, these anomalous coefficients, however, cannot account for many other effects observed experimentally, such as the fast increase of density on the axis of the plasma column due to admission of gas.[2] A common deficiency of such models, which does not allow an adequate description of the aggregate of experimental results, apparently is that only some of the coefficients are increased and the other coefficients either retain their neoclassical values or have appropriate terms deleted in the equations in view of their smallness as compared with the anomalous terms. To eliminate this shortcoming. In selecting the transport coefficients for simulation of the plasma behavior in the tokamak, we must take into account not only the experimental confinement times but also the ratios of the kinetic coefficients from the theory. The latter, however, requires additional hypotheses about the mechanism of anomalous behavior; moreover, the correctness of choice can be confirmed only after comparison of the results of calculations with the experiment.

In this paper we attempt to explain the enhanced transfer in the tokamak by the anomalous electron longitudinal viscosity. Physically, this is equivalent to introducing additional attenuation of the poloidal pulse as compared with neoclassical attenuation. Any mechanism producing this effect can therefore be regarded as a viscosity. As a first step, we obtained a system of transport equations without the heat flux and the thermal diffusion, which can explain, for example, the aforementioned fast increase of the density on the axis of the plasma column.

Let us examine a simple electron-ion plasma. The original equation for the particle flux is the equilibrium equation

$$-\vec{\nabla} p - \vec{\nabla} \pi + en(\mathbf{E} + [\mathbf{v}\mathbf{H}]/c) + \mathbf{R} = 0. \qquad (1)$$

To simplify its derivation, we shall ignore small fluxes, in this case the classical flux and the Pfirsch-Schlüter flux. The former allows us to equate the toroidal and the longitudinal components $E_\phi = E_\parallel$ and $R_\phi = R_\parallel$ and the latter allows us to replace $E_\parallel$ and $R_\parallel$ by their average values at the magnetic surface. (Henceforth, all the values denote average values.) The toroidal projection of Eq. (1), to which ∇_p and ∇_π do not contribute, allows us to express the radial particle flux in terms of the longitudinal components of the frictional force and of the electric field:

$$nv_r = (c/eH_\theta)\,[-neE_\parallel + m_e n\nu_{ei}(v_{i\parallel} - v_{e\parallel})]. \qquad (2)$$

This expression may be represented as a longitudinal Ohm's law, if j/ne is substituted in it for $v_{i\parallel} - v_{e\parallel}$:

$$j_\parallel = \sigma(E_\parallel + v_r H_\theta / c\ ). \qquad (3)$$

The velocities $v_{i\parallel}$ and $v_{e\parallel}$ in Eq. (2) can be determined from the projection of Eq. (1) in the direction of the magnetic field[3]:

$$\begin{aligned} &-\mu_e(v_{e\parallel} - v_{eD}) - enE_\parallel + m_e n\nu_{ei}\,(v_{i\parallel} - v_{e\parallel}) = 0,\\ &-\mu_i\,(v_{i\parallel} - v_{iD}) + enE_\parallel + m_e n\nu_{ei}\,(v_{e\parallel} - v_{i\parallel}) = 0, \end{aligned} \qquad (4)$$

where μ_e and μ_i are the viscosity coefficients divided by $(H_\theta/H_\phi R)^2$ and v_{eD} and v_{iD} are diamagnetic velocities calculated from the poloidal field

$$v_{aD} = \frac{c}{e_a n H_\theta}\left(\frac{\partial p_a}{\partial r} - e_a n E_r\right).$$

The solution of Eq. (4), after substitution in Eqs. (2) and (3), gives the following results:

$$j_\parallel = \frac{\sigma}{1+\gamma}E_\parallel - \frac{\gamma}{1+\gamma}\frac{c}{H_\theta}\frac{\partial p}{\partial r}, \qquad (5)$$

$$nv_r = -\frac{\gamma}{1+\gamma}\,nc\,\frac{E_\parallel}{H_\theta} - \frac{\gamma}{1+\gamma}\,\nu_{ei}\,\rho_{pe}^2\,\frac{1}{p}\,\frac{\partial p}{\partial r}, \qquad (6)$$

where $\rho_{pe}^2 = 2c^2T_e m_e/e^2H_\theta^2$ is the Larmor radius of the poloidal field, $p = p_e + p_i$, $\sigma = ne^2/m_e\nu_{ei}$, $\gamma = \gamma_i\gamma_e/(\gamma_i + \gamma_e)$, $\gamma_a = \mu_a/m_a n\nu_{a\beta}$ is the parameter characterizing the viscosity-to-friction ratio. The first term on the right-hand side of Eq. (5) describes the conduction current and the second term describes the bootstrap current. The corresponding terms in Eq. (6) give the pinch rate of the plasma column and the diffusion flow rate.

Since $\mu_e \sim (m_e/m_i)^{1/2}\mu_i$ is the neoclassical theory, $\gamma^{Hc} \approx \gamma_e$. Using the neoclassical viscosity coefficient of the electrons μ_e[3] (see Fig. 1), we can make the following estimates for γ^{Hc}: in the $(\omega_{be}\epsilon^{3/2} < \nu_e < \omega_{be})$ regime $\gamma^{Hc} \sim \epsilon^2\omega_{be}/\nu_e$ and in the banana

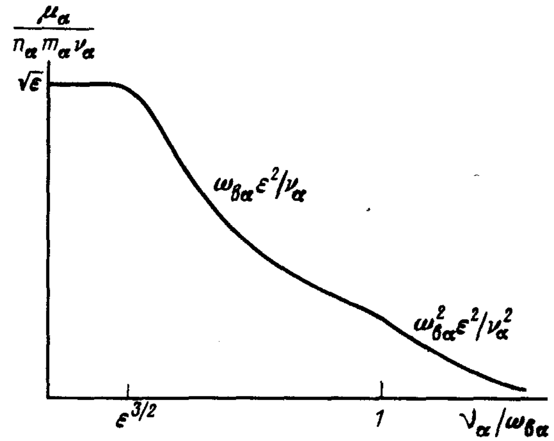

FIG. 1. Dependence of the neoclassical viscosity coefficient μ_α on the collision frequency.

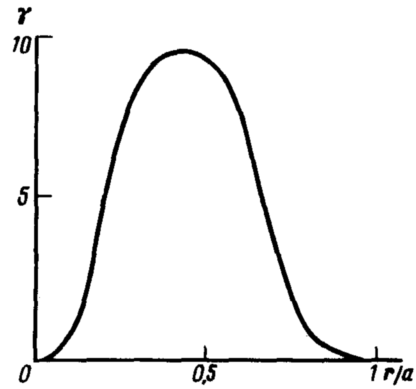

FIG. 2. Dependence of the paramter γ on the radius for characteristic discharge parameters of the T-10 tokamak: $T_e = 1$ keV and $n_e = 5\times10^{13}$ cm^{-3}.

regime $(\nu_e < \omega_{be}\epsilon^{3/2})\gamma^{Hc} \sim \epsilon^{1/2}$, where $\omega_{be} = v_{Te}/qR$ and $\epsilon = a/R \ll 1$. Substituting these γ values in Eqs. (5) and (6), we obtain the well-known expressions for neoclassical particle flux and Ohm's law[1] (we remind that the thermal diffusion and heat flux are disregarded).

Let us now examine the anomalous behavior of electrons. In terms of the hydrodynamic approach described above we can accomplish this by increasing the values of the kinetic coefficients μ_e and ν_{ei}. A variation of ν_{ei}, however, would violate Ohm's law,[3] which is confirmed in the experiments using tokamaks.[4] Therefore, the viscosity coefficient μ_e is the only parameter that can be used to obtain the anomalous transport coefficients.

Let us examine the limiting case. Suppose μ_e is so large that it exceeds the neoclassical ion viscosity coefficient $\mu_e \gg \mu_i^{Hc}$. Thus, the parameter γ, which is determined by the ionic component, exceeds the neoclassical parameter by a factor of $(m_i/m_e)^{1/2}$. We can see that almost in the entire region of the impact parameter, which corresponds to the neoclassical regimes $(\nu_e < \omega_{be})\gamma \doteq \gamma_i^{Hc} \gg 1$ (see Fig. 2). The diffusion coefficient in this case attains the maximum value for the hydrodynamic description, which is equal to the pseudoclassical value $D=\nu_{ei}\rho_{pe}^2$.[5] The pinch of the plasma column, which reaches the value $v_r=cE_\parallel/H_\theta$ at $\gamma\gg 1$, increases proportionally to the increase of the diffusion coefficient. Such pinch rate is entirely adequate to account for the experimentally observed rapid increase of the density at the plasma column's axis. It is interesting to note that in the limiting case $\gamma\gg 1$ the conduction current vanishes; however, if the plasma is in equilibrium ($v_r=0$), then the simple form of the Ohm's law is conserved $j_\parallel=\sigma E_\parallel$ (3).

The plasma confinement time can be reconciled with those experiments for which the pseudoclassical diffusion is too large by selecting the parameter γ. In this case the relations between the transport coefficients in the expressions (5) and (6) are not violated and hence the conclusions reached for the case $\gamma\gg 1$ remain valid. Thus, the observed effects can be described more completely by assuming that the enhanced transport in the tokamak is due to anomalous electron viscosity. Moreover, it limits the range of permissible variation of the diffusion coefficient to the neoclassical and pseudoclassical values, which also corresponds to the available experimental data.

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