Families of particles and compound $SU(5)$ decuplets

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A model in which the $SU(5)$ decuplets of quarks and leptons are compound decuplets is examined. The model requires the existence of three families (generations) of quarks and leptons.

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At present, the existence of families or generations of leptons ($e\nu_e, \mu\nu_\mu\ \tau\nu_\tau$) and quarks ($ud$, cs, $t?b$) has no completely satisfactory explanation. The most successful scheme of unified classification of quarks and leptons—the $SU(5)$ symmetry (1)—examines each family independently. In this paper we attempt to formulate a scheme which requires the existence of three families of particles.

Of the two multiplets of the $SU(5)$ group which unify the quarks and leptons of one family, one multiplet—antiquintet of left-handed particles—is the fundamental multiplet, whereas the second multiplet—decuplet—can be produced by antisymmetrization of two fundamental quintets: $5\times 5 = 10 + 15$. A decuplet can also be constructed in terms of the fundamental fives: $\underset{\sim}{\bar{5}}\times\underset{\sim}{\bar{5}}\times\underset{\sim}{\bar{5}} = (\underset{\sim}{\overline{10}} + \underset{\sim}{\overline{15}})\times\bar{5} = \underset{\sim}{10} + \underset{\sim}{\overline{40}} + \underset{\sim}{\overline{40}} + \underset{\sim}{\overline{35}}$. Here the decuplet is a totally antisymmetric combination of three antiquintets. This group family can be given a physical meaning, if we assume the existence of new particles $Q_{\alpha i}$ ($\alpha = 1,\ldots,\beta$, $i = 1,2,3$) for which the index i, which includes three values, numbers the states of the unbroken symmetry that is similar to the ordinary color group. We shall call these particles "quints" and the new symmetry group the "age group". The confinement radius of the "age group" is very small, and we assume that the quarks and leptons in the decuplet are the bound states of the $Q_{\alpha i}$ particles. We shall also assume that the left-handed decuplet $\underset{\sim}{10}$ is comprised of massless quints with right-handed helicity:

$$\Psi_L^{\lambda\mu}(\mathrm{p}) \sim Q_{\alpha i,R}(\mathrm{p}_1)Q_{\beta j,R}(\mathrm{p}_2)Q_{\gamma k,R}(\mathrm{p}_3)\epsilon^{\alpha\beta\gamma\lambda\mu}\epsilon^{ijk}. \tag{1}$$

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It is clear that in this case the momentum of the compound particle **p**, roughly speaking, should be oriented in the direction of the momentum of one of the quints (for example, $\mathbf{p}_1$) and the other two momenta should be oriented in the opposite direction. This may seem strange, since it would be easier to assume the existence of a massless particle comprised of three massless particles moving in the same direction. We should bear in mind, however, that in a dynamical treatment the particles may turn out to be in the states with a negative energy and then the zero mass of the compound particle $(p_1 + p_2 + p_3)^2 = 0$ may be obtained from the massless quints: $p_1^2 = p_2^2 = p_3^2 = 0$ like in the case of the free particles with a positive energy, which move in the same direction.[1)]

The $u + d$ quark pair contains the same quints as the $\widetilde{u} + e^+$ pair. Thus, the $u + d \to \tilde{u}^+ e^+$ reaction due to a simple rearrangement of quints is possible. Hence, taking into account that the cross section of this reaction is $\sigma \sim r^4 s$ (r is the confinement radius of the age group and $\sqrt{s}$ is the energy of the quarks), we can easily determine the lifetime of the proton $T_p \sim R^5/r^4$, where R is the radius of the ordinary confinement. Assuming that $r = 1/M$, we have from the experimental limitation of the proton's lifetime $M \geqslant 10^{15}$ GeV, which coincides in order of magnitude with the unification mass of the $SU(5)$ symmetry.

Since the full scale of the masses is so large, we might ask why the observed fermions do not have a mass of $\sim r^{-1} \sim 10^{15}$ GeV? This mass must be connected with the production of the condensate of quints $\langle Q_{\alpha i,R}\, Q_l^{\beta i} \rangle \neq 0$ in a similar way as the generation of nucleon mass is connected with the production of the condensate of quarks $\langle \bar{u}_R^i\, u_L^i \rangle = \langle \bar{d}_R^i\, d_L^i \rangle \neq 0$. In the case under consideration, however, the condensate inevitably leads to a breakdown of the electromagnetic and color groups, since the left-handed and right-handed quints belong to different $SU(5)$ representations. We assume, therefore, that such condensation does not occur. Thus, the chiral symmetry (independent phase transformation of the left-handed and right-handed particles), remains unbroken which accounts for the absence of the mass of scale 10^{15} GeV of the fermions.

By introducing quints into the theory, we thereby introduce the Adler anomaly if each of the families is examined separately. The anomaly associated with the three *quintets* of the left-handed quints $Q_L^{\alpha i}$ ($i = 1,2,3$) can be canceled out by the three *antiquintets* of the ordinary left-handed particles that belong to different families. For this, however, the quints must not have a degree of freedom that links them to a given family. On the other hand, there are nonetheless three left-handed decuplets rather than one. We assume that the decuplets—bound states of three quints—may differ by a certain "principal quantum number" n. Thus, in view of what was said above all the states with different n remain massless spiral states up to the inclusion of the phenomenological Higgs couplings that couple the compound tens with the fives or the tens with each other. The Higgs mechanism leads to the production of three observable families of massive fermions plus additional massless decuplets, if only the principal quantum number does not equal to exactly three. *All* the other three-quint states belonging to other $SU(5)$ group representations also remain massless in a similar way. Like the "excessive" decuplets, all of them do not have partners to form a mass: particles with the same electric and color charge but with opposite helicity (the only exception is the left-handed helical state constructed from three neutral quints which could produce a massive fermion along with a right-handed antineutrino). Thus, the observed quintets and decuplets exhaust all the possible massive states. As for the massless but charged particles, we simply assume that they cannot be observed. Here we encounter a complicated, dynamic problem which we shall not discuss in this paper. We note that the situation is different for mesons built from quints. There are no constraints on helicity for the scalar mesons (15-tuplet) of the type $M^{\alpha\beta} \sim (Q_L^{\alpha i} C Q_L^{\beta i})$, and hence we expect that their mass is $\sim 10^{15}$ GeV. In principle, a condensate can be produced for the vector mesons. $V_\beta^\alpha \sim (Q_L^{\alpha i} C \gamma_\mu Q_{\beta i, R})$, because they are transformed according to the real $SU(5)$ representations.

Since $\mu_{\bar{R}}$ is a "radial" perturbation of $e_{\bar{R}}$, generally, the electromagnetic transition between them is allowed. This transition is produced by the current of the type: $a(q^2)\bar{\mu}_R(q^2\gamma_j - \hat{q}q_i)\, l_R$, where $a(0) \approx r^2 \approx M^{-2}$. It follows from this that the $\mu \to e + \gamma$ decay is in fact forbidden, and for the $\mu \to 3e$ $\Gamma(\mu \to 3e)/\Gamma(\mu \to e\nu_e\, \nu_\mu) \approx (M_w/M)^4 \lesssim 10^{-52}$.

Thus, if we examine the quintet $SU(5)$ representations, then the left-handed particles—quints—are the triplets of the age group but do not have an index indicating their affiliation with a specific family. If the family group (which must be strongly violated), like the age group, is a $SU(2)$ group, then the quints are the singlets of this group. Conversely, the ordinary right-handed quintets of the $SU(5)$ group are the singlets of the age group and the triplets of the family group. Thus, all the elementary fermions are classified according to the groups

$$SU(5) \times SU(2)_L^a \times SU(2)^f_L \tag{2}$$

and belong to two representations: (5, 3, 1) and (5, 1, 3). L and R switch places for antiparticles that are transformed according to the antiquintet representation. Of the two $SU(2)$ groups the $SU(2)$ family group is strongly violated and the $SU(2)^a$ age group remains exact. The advantage of using the $SU(2)$ groups [rather than $SU(3)$] is obvious: we thus get rid of the Adler anomalies associated with their gauge bosons.

The direct product of the chiral groups $SU(2)_L^a \times SU(2)_R^f$ is isomorphous to the $SO(4)$ group. Therefore, we can write instead of Eq. (2):

$$SU(5) \times SO(4) \tag{3}$$

and state that all the elementary fermions $Q_{\mu\nu}^\alpha$ belong to the $(\underset{\sim}{5}, \underset{\sim}{6})$ representation of these groups, where 6 is the antisymmetric tensor representation of the $SO(4)$ group. We note that although the $Q_{\mu\nu}^\alpha$ spinors are four-component spinors, they satisfy the relation arising from the chiral nature of the $SU(2)$ groups:

$$Q_{\mu\nu}^\alpha = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\,\gamma_5 Q_{\rho\sigma}^\alpha \ . \tag{4}$$

In this paper we shall not describe the Higgs sector. We shall explain, however, how the $SO(4)$ group can be naturally broken down to $SU(2)^a$. For this we can obviously use, for example, the Higgs field which is transformed as a right-handed spinor or self-dual tensor according to the $O(4)$ group. In the first case, all the gauge bosons of the group of families acquire the same mass and in the second case two bosons acquire the same mass and one boson remains massless (and thus must become massive due to the other Higgs couplings). A breakdown of the $O(4)$ group automatically breaks down ordinary parity, because the gauge bosons, which interact with the left-handed fermions remain massless and those which interact with the right-handed fermions become massive. Before this occurs, all the currents of the $SU(5)$ group are pure vector currents. However, the left- and right-hand components of the corresponding spinors, on the one hand, are quints and, on the other, are particles that belong to different families.

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[1] H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32**, 438 (1974).