Mass spectrum of mesons in the quasi-potential approach.

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The masses of J/ψ and γ mesons are calculated by using the the quasi-potential Logunov–Tavkhelidze equation. The potential was chosen in the form $V(r) = \sigma r$. A good agreement with the experiment is obtained.

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Lately, the mass spectrum of the J/ψ and γ particles has been investigated intensively by using the nonrelativistic Schrödinger equation (see review article[1]). The mesons in this case are regarded as bound states of the quark and antiquark and the potential is assumed to be increasing as $r \to \infty$.

In this paper the mass spectrum of the S states of the vector mesons is calculated on the basis of the relativistic, three-dimensional, quasi-potential Logunov–Tavkhelidze equation

$$(E^2 - m^2 - \hat{\mathbf{p}}^2)\, \Phi(\mathbf{r}) = - \frac{m^2 V(r)}{\sqrt{m^2 + \hat{\mathbf{p}}^2}}\, \Phi(\mathbf{r}) \qquad \hbar = c = 1. \qquad (1)$$

Here E is the total energy of a quark, m is the mass of a quark, and $\hat{p} = -i\nabla$.

TABLE I.

	$M_{c\bar{c}}$, GeV		$M_{b\bar{b}}$, GeV	
n	Experiment	Theory	Experiment	Theory
1	3.095	3.095	9.46	9.46
2	3.686	3.686	10.01	10.01
3	4.04	4.082	10.38	10.43
4	4.41	4.391	–	10.78
5	–	4.647	–	11.09
6	–	4.868	–	11.37

We shall choose the potential in Eq. (1) in the form $V(r) = \sigma r$ and substitute $\Phi(\mathbf{r}) = (\chi_l(r)/r)Y_{lm}(\theta, \phi))$. Thus, for $l = 0$, we have

$$\left(E^2 - m^2 + \frac{d^2}{dr^2}\right)\chi_0(r) = -\frac{m^2 \sigma r}{\sqrt{m^2 - (d^2/dr^2)}}\chi_0(r). \tag{2}$$

In Eq. (2) we shall go over to the Fourier transform of the $\chi_0(r)$ function:

$$\tilde{\chi}_0(p) = \int_{-\infty}^{+\infty} e^{ipr}\chi(r)\,dr. \tag{3}$$

For $\tilde{\chi}_0(p)$ we obtain a first-order differential equation:

$$-im^2\sigma \frac{d}{dp}\ln\tilde{\chi}_0(p) = \sqrt{m^2 + p^2}(E^2 - m^2 - p^2). \tag{4}$$

$\tilde{\chi}_0(p)$ can be calculated by simple integration. To obtain the quantization condition, we shall go from $\tilde{\chi}_0(p)$ to $\chi_0(r)$ using the inverse Fourier transformation and set $\chi_0(0) = 0$. We obtain

$$\int_0^\infty dp \cos\left\{\frac{1}{2m^2\sigma}\left[p\sqrt{p^2 + m^2}\left(E^2 - \frac{5}{4}m^2 - \frac{p^2}{2}\right)\right.\right.$$

$$+ \frac{1}{2} m^2 \left(E^2 - \frac{3}{4} m^2\right) \text{arc sinh} \frac{p}{m} \Bigg]\Bigg\} = 0. \qquad (5)$$

Integration in Eq. (5) can be performed by using the stationary-phase method, which gives the following approximate quantization condition:

$$E_n \sqrt{E_n^2 + m^2} \left(E_n^2 - \frac{3}{2} m^2\right) + 2m^2 \left(E_n^2 - \frac{3}{4} m^2\right) \text{arc cosh} \frac{E_n}{m}$$
$$= 4\pi m^2 \sigma \left(n + 3/4\right); \quad \pi \left(n + 3/4\right) >> 1 \qquad (6)$$

where n is positive integer.

According to Ref. 2, we assume that the mass of the bound state $q\bar{q}$ is equal to $M_{q\bar{q}} = 2E_q$. The parameters m and σ are determined from the masses of the first two levels with quantum numbers $n = 1$ and 2. Thus, for the J/ψ particles we obtain $\sigma_c = 0.60$ GeV^2 and $m_c = 0.961$ GeV and for the γ particles we have $\sigma_b = 0.45$ GeV^2 and $m_b = 4.34$ GeV. The mass spectra in Table I, which were calculated with the help of these parameters for different values of the principal quantum number n, are in good agreement with the experiment. [The quantization condition (5) gives a 10% variation of the parameters σ and m.] Moreover, Table I gives the theoretical predictions obtained within the framework of the model examined above.

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[1]C. Quigg and Y. Rosner, Fermilab Pub. 79/22 THY, 1979.
[2]V. G. Kadyshevskiĭ and A. N. Tavkhelidze, A book entitled "Problemy teoreticheskoĭ fiziki," (Problems in Theoretical Physics), Nauka, M., 1969, p. 261.