Fan instability and anomalous ion heating

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The mechanism of anomalous ion heating observed in tokamaks with moderate densities is considered.

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As is well-known,[1] moderate-density regimes ($n_e \lesssim 10^{13}$ cm^{-3}) in tokamaks are, as a rule, accompanied by the development of the so-called "fan" instability, which manifests itself in periodic variations of the macroscopic characteristics of a plasma. The instability is caused by runaway electrons, which lead, under certain conditions, to a buildup of Langmuir oscillations due to the anomalous Doppler effect.[2] Existing theory[3] provides a self-similar description of the basic experimentally-observed effects of this instability, excluding the anomalous ion heating. We recall that as was shown in the experiments, each burst of instability is accompanied by intense ion heating; moreover, the amount of energy transmitted to ions is comparable to the increase in the transverse energy of the beam electrons.

It is known that the "fan" instability leads to excitation of high-frequency oscillations with $\omega \lesssim \omega_{pe}$, which fail to directly couple to the ions. Hence, the oscillation energy may be coupled to the ions only by means of nonlinear processes that lead to wave energy transfer into a frequency region on the order of ω_{pi}. We start by estimating the fraction of beam energy transferred to oscillations. Inasmuch as each particle loses energy $\Delta\epsilon \sim (\omega/\omega_{Be})\epsilon$ in the course a reversal of direction in velocity,[2] the energy converted to oscillations during each instability burst is $W \lesssim (\omega_{pe}/\omega_{Be})E_b$. If we assume that as a rule $E_b \sim n_e T_e$ in discharges with the "fan" instability, we obtain the limit $\dfrac{W}{n_e T_e} \lesssim \dfrac{\omega_{pe}}{\omega_{Be}} \lesssim 1$. Clearly, nonlinear effects under these conditions may play a significant role. Below we shall consider a single nonlinear mechanism (the one having the lowest threshold with respect to $W/n_e T_e$)—induced scattering of Langmuir waves by ions.[4] In each act of induced scattering the wave frequency varies by $\Delta\omega \lesssim \omega_{pi} \ll \omega_{pe}$; the energy $\hbar\Delta\omega$ is transferred to the resonant ions. It follows that a Langmuir wave must undergo $N \sim \omega/\Delta\omega$ scattering events in order to transfer a considerable portion of its energy to the ions.

The wave simultaneously loses its energy due to Landau damping and electron-ion collisions. This energy goes into heating the electron component. The ion heating effectiveness will naturally be higher the greater the rate of the nonlinear scattering process (which is proportional to the wave energy density $W/n_e T_e$). The induced scattering process is quantitatively described by the following system of integro-differential equations:

$$\frac{\partial n_{\mathbf{k}}}{\partial t} + \gamma_{\mathbf{k}} n_{\mathbf{k}} = \int d\mathbf{k}' T_{\mathbf{k}\mathbf{k}'} n_{\mathbf{k}} n_{\mathbf{k}'}, \tag{1}$$

$$\frac{\partial n_e}{\partial t} + \gamma_e n_e = \Gamma n_e - n_e \int d\mathbf{k}' T_{e\mathbf{k}'} n_{\mathbf{k}'} \quad . \tag{2}$$

Here $n_{\mathbf{k}}$ is the spectral density of the number of Langmuir wave quanta generated in the course of nonlinear energy transfer; n_e is the density of the number of "primary" wave quanta excited by an instability; $\gamma_{\mathbf{k}}$ is the linear damping rate of Langmuir waves; $\Gamma \sim \frac{\omega_{pe}^2}{\omega_{Be}} \frac{n_b}{n_e}$ is the growth rate of the primary Langmuir waves; $T_{\mathbf{k}\mathbf{k}'}$, is the matrix element of nonlinear coupling.

Below, for the sake of simplicity, we shall assume that the electron beam excites a wave packet which is monochromatic with respect to ω and k, such that

$$N n_e = N_e \delta(\omega - \omega_0) \frac{\delta(k - k_0)}{2\pi k^2};$$

this assumption does not change the order of magnitude of the results. Moreover, Eqs. (1) and (2) may be simplified by assuming that the wave frequency in each scattering event varies negligibly. This leads in Eq. (1) to the following differential approximation:

$$\frac{\partial}{\partial t} n(x, k) + \gamma_k n(x, k) = \int 2\pi k'^2 dk' T_{kk'} (\Delta x)^2 n(x, k) \frac{\partial}{\partial x} n(x, k') \tag{3}$$

$$+ \Delta x T_{kk'} n(x_0) N_e \delta(x - x_0),$$

$$\frac{\partial N_e}{\partial t} = \Gamma N_e - \gamma_e N_e - \Delta x \int 2\pi k'^2 dk' n(k', x_0) N_e T_{kk'}(x_0), \tag{4}$$

where

$$x = \frac{\omega}{\omega_{pe}}; \; \Delta x \approx \frac{k v_{Ti}}{\omega_{pe}} \lesssim \frac{\omega_{pi}}{\omega_{pe}}$$

is the characteristic size of the spectral transfer rate. In the following discussion the explicit form of the matrix element $T_{kk'}$, will not be necessary. We note only that analytical and numerical calculations show[4,5] that the process of spectral transfer, described by Eqs. (3) and (4), rapidly leads to generation of a narrow (with respect to $k \approx k_m$) spectrum; moreover, the characteristic quantity k_m is such that the basic contribution to damping of the "secondary" Langmuir waves derives from electron-ion collisions and not from the Landau damping. Therefore, we shall assume below that

$$n(x,k) = n(x) \frac{\delta(k - k_m)}{2\pi k^2} \text{ and } \gamma_k \approx \nu_{ei}.$$

It also appears that $T_{kk'}$, is weakly dependent on x. In this case Eqs. (3) and (4) may be rewritten as follows:

$$\frac{\partial n}{\partial t} = -\nu_{ei} n + T_0 (\Delta x)^2 n \frac{\partial n}{\partial x} + T_0 \Delta x n N_e \delta(x - x_0), \tag{5}$$

$$\frac{\partial N_e}{\partial t} = (\Gamma - \nu_{ei}) N_e - T_0 \Delta x N_e n(x_0). \tag{6}$$

The system (5)–(6) adequately describes the behavior of the Langmuir waves. At $t\Gamma \gg 1$, it may be assumed that $\partial N_e/\partial t = 0$; thus, it follows from Eq. (6) that

$$n(x_0) \equiv n_0 = \frac{\Gamma - \nu_{ei}}{T_0 \Delta x}. \tag{7}$$

We shall now integrate Eq. (5) with respect to x over a narrow strip near $x = x_0$; assuming that $n(x_0 + \delta) \equiv 0$, we get:

$$N_e = n_0 \Delta x = \frac{\Gamma - \nu_{ei}}{T_0}, \tag{8}$$

The value of Eq. (7) constitutes a boundary condition for Eq. (5). We shall initially assume that the dissipation is small. In this case Eq. (5) describes wave breaking, since it is identical to the nonlinear equation of motion of a liquid. Inasmuch as the characteristic time for wave steepening $\tau_1 \sim (T_0 \Delta x n_0)^{-1}$ is much shorter than the time τ_2 required for a wave to transit the distance $x \sim x_0$ (during which the wave frequency changes to ω_{pi}), $\tau_2 \sim \tau_1/\Delta x$, we may at once seek a solution in the region in the form of a step function:

$$n(x) = n_0 \theta(x - x_f(t)) = n_0 \times \begin{cases} 1 & \text{for } x > x_f \\ 0 & \text{for } x \leqslant x_f \end{cases}. \tag{9}$$

To find the time dependence of the front coordinate x_f, we shall integrate Eq. (5) with respect to x from $x_1 = 0$ to $x_2 = x_0 + \delta$:

$$x_f = x_0 - \frac{T_0 (\Delta x)^2 n_0}{2} t = x_0 - \frac{(\Gamma - \nu_{ei}) \Delta x}{2} t. \tag{10}$$

Subsequently, after the time $\tau \sim \tau_2 \approx 5\omega_{pe}/(\Gamma - \nu_{ei})\omega_{pi}$ the "primary" (excited by the beam) Langmuir wave frequency will decrease to $\omega \ll \omega_0$, i.e., it will transfer a substantial portion of its energy to the ions. We hasten to note that the value of τ_2 is independent of the magnitude of the matrix element $T_{kk'}$.

We will now estimate the amount of energy lost by a wave due to electron-ion collisions. Assuming that the collisions remove a small fraction of energy (this assumption is confirmed by the results), Eq. (5) yields the following:

$$\frac{(\Delta W)_{\nu_{ei}}}{W(x_0)} \approx \frac{\nu_{ei}}{\Gamma - \nu_{ei}} \frac{5\omega_{pe}}{\omega_{pi}} . \tag{11}$$

Under real experimental conditions, $\frac{(\Delta W)_{\nu_{ei}}}{W(x_0)} \lesssim 10^{-2}$. This leads to a conclusion that practically the entire energy of oscillations that have been excited by the "fan" instability, goes into heating resonant ions. As was mentioned above, $W \sim E_b \omega_{pe}/\omega_{Be}$, so that $\sim \omega_{pe}/\omega_{Be}$ of the total beam energy is spent on the heating of ions, a result also observed in experiments.

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