Flow birefringence in a superfluid

A. F. Andreev
Institute of Physical Problems, USSR Academy of Sciences

(Submitted 29 December 1979)
Pis'ma Zh. Eksp. Teor. Fiz. **31**, No. 3, 191–193 (5 February 1980)

It is shown that an optical anisotropy, proportional to the square of the relative velocity, appears in a superfluid liquid in the presence of a relative motion of the normal and superfluid components.

PACS numbers: 42.10.Qj, 67.40. – w

The uniform flow of normal liquids does not lead to the appearance of optical anisotropy in them since in the nonrelativistic case the velocity, in itself, does not of course alter the dielectric constant. Flow birefringence (the Maxwell effect) appears in normal liquids (see Ref. 1) only in the presence of velocity gradients, i.e., it is a dissipative effect.

In a superfluid liquid two types of macroscopic motion are possible, with velocities corresponding to the velocities of the superfluid $\mathbf{v}_s$ and normal $\mathbf{v}_n$ components.

 0021-3640/80/030175-03$00.60

Therefore the dielectric constant tensor can depend on the relative velocity $\mathbf{w} = \mathbf{v}_n - \mathbf{v}_s$. In this way an optical anisotropy, defined by the relation

$$\delta\epsilon_{ik} = \lambda w_i w_k \tag{1}$$

can appear in a homogeneous liquid that is in thermodynamic equilibrium. Here $\delta\epsilon_{ik}$ is the nonscalar part of the dielectric constant, λ is a constant, more precisely a function of the temperature and pressure. Thus, birefringence appears, i.e., a difference in the dielectric constant for light, polarized along the direction of the relative velocity $\mathbf{w}$ and in the perpendicular direction, equal to λw^2.

The birefringence constant λ can be calculated in the following manner. Let us first consider the region of comparatively low temperatures, in which the primary type of elementary excitations in a liquid is phonons. Their presence can be described as the appearance of fluctuations in the density, the characteristic wavelength of which is large compared to the interatomic distance, but small compared to the wavelength of light (at not too low temperatures). The Fourier spatial harmonics of the density $\rho_{\mathbf{k}}$ are expressed here in terms of the production $a_{\mathbf{k}}^+$ and annihilation $a_{\mathbf{k}}$ operators of phonons with momentum $\mathbf{k}$ by the well-known relation

$$\rho_{\mathbf{k}} = \left(\frac{\rho k^2}{2\omega_k}\right)^{1/2} (a_{\mathbf{k}} + a^+_{-\mathbf{k}}), \tag{2}$$

where ρ is the density of the liquid, $\omega_{\mathbf{k}} = sk$, s is the velocity of sound.

In the general case when the momentum direction distribution of the phonons is not isotropic, the liquid is characterized by a nonisotropic dielectric constant. In view of the macroscopic character of the problem, the nonscalar part $\delta\epsilon_{ik}$ of the dielectric constant can be calculated exactly. The result can be represented in the form (see Ref. 2):

$$\delta\epsilon_{ik} = -\frac{1}{\epsilon}\left(\frac{\partial\epsilon}{\partial\rho}\right)^2 \frac{1}{V}\sum_k \frac{k_i k_k}{k^2} < |\rho_k|^2 >, \tag{3}$$

where ϵ is the isotropic part of the dielectric constant, V is the normalizing volume. After substitution of (2) into (3) we obtain an expression for $\delta\epsilon_{ik}$ in terms of the phonon distribution function $n_{\mathbf{k}}$:

$$\delta\epsilon_{ik} = -\frac{\rho}{\epsilon}\left(\frac{\partial\epsilon}{\partial\rho}\right)^2 \int \frac{d^3k}{(2\pi)^3} \frac{k_i k_k}{\omega_k} n_{\mathbf{k}} . \tag{4}$$

When superfluid and normal motions with velocities $\mathbf{v}_s$ and $\mathbf{v}_n$ are present in the liquid, the distribution function is equal to $n_0(\omega_k - \mathbf{kw})$, where $n_0(\omega)$ is the Planck distribution function. Substituting this function into formula (4) and expanding in powers of $\mathbf{w}$ up to second-order terms, we obtain

$$\delta\epsilon_{ik} = -\frac{\rho\rho_n}{\epsilon s^2}\left(\frac{\partial\epsilon}{\partial\rho}\right)^2 w_i w_k , \tag{5}$$

where ρ_n is the phonon part of the density of the normal component. In liquid helium the dielectric constant is close to unity and to a high degree of accuracy the derivative $\partial\epsilon/\partial\rho$ is equal to $(\epsilon-1)/\rho$. Therefore in place of (5) we have

$$\delta\epsilon_{ik} = -(\epsilon-1)^2 \frac{\rho_n}{\rho} \frac{w_i w_k}{s^2}. \tag{6}$$

In the high-temperature roton region a formula of the form

$$\delta\epsilon_{ik} = -\frac{(\epsilon-1)^2}{\rho} a \int \frac{d^3k}{(2\pi)^3} \nu_i \nu_k n_{\mathbf{k}}, \tag{7}$$

analogous to (4), can be used, where $\nu = \mathbf{k}/k$, $n_{\mathbf{k}}$ is the roton distribution function, a is some phenomenological constant. As before, from (7) we obtain

$$\delta\epsilon_{ik} = -(\epsilon-1)^2 \frac{\rho_n}{\rho} \frac{a}{5T} w_i w_k, \tag{8}$$

where ρ_n is the roton normal density. The hydrodynamic formula (4) can be used to evaluate the constant a by taking $|\mathbf{k}| = p_0$, $\omega_{\mathbf{k}} = \Delta$ for rotons, where p_0, Δ are the roton momentum and energy. In this way we obtain $a = p_0^2/\Delta$.

It follows from (8) that for $\rho_n \sim \rho$ the difference in the dielectric constants for light, polarized along and perpendicular to w, is equal, in order of magnitude, to $(\epsilon-1)^2(w/v_L)^2 \sim 3\times10^{-3}(w/v_L)^2$, v_L is the critical Landau velocity. This value, obviously, can be found by completely experimental means.

In conclusion let us note that the appearance of the density anisotropy of the normal component of the liquid in the external electric field $\mathbf{E}$ is thermodynamically related to the effect discussed here. In fact, it follows from the thermodynamic identity $dU = -(1/4\pi)\mathbf{D}d\mathbf{E} - \mathbf{p}\,d\mathbf{w}$, where $D_i = \epsilon_{ik}E_k$, $\mathbf{p}$ is the momentum of the relative velocity of the normal and superfluid components, and Eq. (1) that the density of the normal component $\rho_{ik}^{(n)} = \partial p_i/\partial w_k$ contains a nonscalar part equal to $(\lambda/4\pi)E_iE_k$.

[1] L. D. Landau and E. M. Lifshits, Elektrodinamika sploshnykh sred (Electrodynamics of Continuous Media), Fizmatgiz Press, Moscow, 1959, p. 81.

[2] A. F. Andreev, Pis'ma Zh. Eksp. Teor. Fiz. **19**, 713 (1974) [JETP Lett. **19**, 368 (1974)].

Translated by Eugene R. Heath. Edited by David Book