Influence of pionization on hard jets

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A modification of the equations of evolution of hard jets in hadronic processes is proposed, accounting for the formation of soft (pionization) quark-antiquark pairs during the motion of hard quarks.

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The process of e^+e^- annihilation at high energies leads to the creation of a quark-antiquark pair, which in the course of separation emits gluons that generate new quark-antiquark pairs and gluons. It is usually assumed that hard jets along the direction of the motion of the primary quark and antiquark arise precisely in this manner. The behavior of the jets is described by equations obtained in Refs. 1–3. They take into account all possible chromodynamic vertices of the emission of gluons and the creation of pairs. These processes are hard since the energy at a vertex is divided on the average equally among the newly formed components of the jet.

We would like to point out that, in addition to the hard processes, there should exist soft pair-production processes. They should arise because as the jets diverge there must be created, in the space between them, a strong quark-gluon field, which would restrict the possibility for colored objects to move to a great distance; this field should grow stronger with distance. The real possibility for the process to take place is due to the fact that the growth of the field is limited by the formation of new quark-antiquark pairs compensating for the color degrees of freedom. The mechanism of creation of such pairs (see, e.g., Ref. 4) is connected with the unsolved problem of confinement in quantum chromodynamics. It is conceivable, however, that its effect on the basic hard process is reduced to the quarks and gluons in the jets slowly "gliding" down the energy scale because of the necessity of production of "soft" (pionization) pairs. Without going into the details of production of these pairs in studies of the hard process, we can take into account their role by assuming that they will affect the quark-gluon jets in a manner analogous to the influence exercised by ionization processes on the behavior of the usual electron-photon shower. The analogy is reinforced by the fact that the equations of jet evolution considered earlier[1-3] are strongly similar to the equations of electromagnetic cascade processes disregarding ionization. Making use of the picture described above, we will write the equations of evolution of jets incorporating the effect exerted on them by pionization processes as follows:

$$\frac{\partial D^q(E,Y)}{\partial Y} = \int_E^\infty \frac{dE'}{E'} P^{qq}(E,E')D^q(E',Y) + 2n_f \int_E^\infty \frac{dE'}{E'} P^{qG}(E,E')D^G(E',Y) - \int_0^E \frac{dE'}{E} P^{qq}(E',E)D^q(E,Y) \tag{1}$$

$$+\ \beta_q \frac{\partial D^q(E,Y)}{\partial E}$$

$$\frac{\partial D^G(E,Y)}{\partial Y} = \int_E^\infty \frac{dE'}{E'} P^{Gq}(E,E')D^q(E,Y) + 2\int_E^\infty \frac{dE'}{E'} P^{GG}(E,E')D^G(E',Y)$$

$$-\int_0^E \frac{dE'}{E} P^{GG}(E',E)D^G(E,Y) - n_f \int_0^E \frac{dE'}{E} P^{qG}(E',E)D^G(E,Y) + \beta_G \frac{\partial D^G(E,Y)}{\partial Y}. \quad (2)$$

Here D^a are the functions of the distribution of quarks ($a \approx q$) and gluons ($a = G$) in the energies E and the "depth"

$$Y = \frac{1}{2\pi b} \ln \frac{\ln Q^2/\Lambda^2}{\ln k^2/\Lambda^2}\ ;\ b = \frac{33-2n_f}{12\pi}\ ,\ n_f$$

is the number of flavors, Q^2 and k^2 are the initial and "current" mass in the jet, and Λ is the cutoff parameter, and P^{ab} are the transition probabilities $b \to a$:

$$P^{qq} = c_F \frac{1+x^2}{1-x},\quad P^{qG} = \frac{1}{2}(x^2 + (1-x)^2);\quad P^{Gq} = c_F \frac{1+(1-x)^2}{x^2}\ ;$$

$$P^{GG} = c_V x(1-x)\left[1 + \frac{1}{x^2} + \frac{1}{(1-x)^2}\right];\qquad x = E'/E,\quad c_V = 3,\quad c_F = 4/3.$$

The β_a are pionization parameters that will be discussed later. The last terms in Eqs. (1) and (2) describe the outflow of energy into the pionization component. Actually, they are the only elements that distinguish the system of equations (1) and (2) from the equations given in Ref. 3. The system can be solved by the same method that is used in the theory of electron-photon cascades. After the double Laplace-Mellin transformation

$$M^a_{n,\lambda} = \int_0^\infty dY e^{-\lambda Y} \int_0^1 d\zeta\, \zeta^{n-1} D^a(\zeta, Y) \quad (3)$$

($\zeta = E/E_0$; E is the primary energy), the system of Eqs. (1) and (2) appears as

$$\sum_b (\lambda\delta_{ab} - A^{ab}_n) M^b_{n,\lambda} + \frac{\beta_a}{E_0}(n-1) M^a_{n-1,\lambda} = M^a_n(0), \quad (4)$$

where

$$M^a_n(Y) = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} e^{\lambda Y} M^a_{n,\lambda}\, d\lambda\ ;$$

$$A_n^{qq} = \int_0^1 dx(x^{n-1} - 1)\, P^{qq}(x);$$

$$A_n^{qG} = 2n_f \int_0^1 dx\, x^{n-1}\, P^{qG}(x);$$

$$A_n^{Gq} = \int_0^1 dx\, x^{n-1}\, P^{Gq}(x);$$

$$A_n^{GG} = \int_0^1 dx\, [(2x^{n-1} - 1)\, P^{GG}(x) - n_f P^{qG}(x)]. \qquad (5)$$

Most active in the process of jet development is the gluon component. It gives large multiplicities and the softest spectra (see, e.g., Refs. 5 and 6). It is reasonable, therefore, to begin with incorporating the effects of pionization on gluons, and simplify the system of difference equations (4), setting $\beta_q = 0$, $\beta_G \equiv \beta$. The resulting consequences can be very easily demonstrated with reference to a gluon jet, solving system (4) for initial conditions.

$M_n^G(0) = 1$, $M_n^q(0) = 0$.

In this case it is possible to use the well known methods of calculation of electromagnetic cascade processes.[7] As a result we obtain[1] that the quark-gluon cascade whose behavior is described by equations system (4) achieves the maximum development in the "depth" of the gluon jet:

$$Y_m \approx 0.22 \ln E_0/\beta \qquad (E_0/\beta \gg 1), \qquad (6)$$

the number of gluons in the jet at the maximum being

$$\langle n_G \rangle_{G_j} \approx \frac{1.1}{\sqrt{\ln E_0/\beta}} \frac{E_0}{\beta}. \qquad (7)$$

Equations (6) and (7) differ from the corresponding formulas for the electron-photon shower [see (11.17) in Ref. 7] solely by the numerical coefficients, which show that a quark-gluon shower develops with a greater intensity (given the same β!).

It should be noted, however, that the problem of the parameter β does not have an equally immediate solution. The value of β determines the energy lost by a gluon per unit variation of Y in the course of evolution of the jet. It is closely linked with the particular confinement model being used. Without going into assumptions concerning the details of the confinement mechanism, we will discuss various possibilities of choosing β in purely phenomenological terms.

First of all, resuming the analogy with the electron-photon cascade, let us consider the case of $\beta = \text{const}$. The shower will develop vigorously with increasing energy E_0. However, the maximum lies at masses close to Λ^2:

$$\frac{k^2}{\Lambda^2} \approx 1 + \frac{\beta}{E_0} \ln \frac{Q^2}{\Lambda^2} \qquad (8)$$

so that the chromodynamic constant is very large and the validity of Eqs. (1) and (2) in this region becomes questionable.

However, if we assume that in a unit interval of variation of Y the pionization component receives a given fraction (γ) of the Feynman energy variable $x \approx E/E_0$, then $\beta = \gamma E_0$. In this extreme case the shower will contain a finite number of "shower" particles [see (7)]. The evolution will virtually cease at masses

$$k^2 \approx \Lambda^2(Q^2/\Lambda^2)^\gamma. \tag{9}$$

The case that is usually considered[5,6] in which the evolution of the jet is assumed to end at fixed masses k^2 corresponds to a parameter of the type $\beta \sim E/\ln E_0$, even though formally in the equations used there $\beta = 0$. However, elimination of the infrared divergences (in the results obtained at $\beta = 0$) by cutoff[5] effectively reduces to an indirect inclusion of pionization in a certain special fashion.

The advantage of Eqs. (1) and (2) proposed here lies in the absence of such divergences and a possibility of further specifying the parameter β in analyzing various confinement models. A comparison of equations of type (7) with the experimental data on the multiplicity may be of help in selecting a realistic model.

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[1)] We do not give the final expressions which are similar to those for electron-photon showers (see Ref. 7) and are very cumbersome; they are to appear in a subsequent publication.

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