Spectrum of electron-ripplon system on the surface of liquid helium in the presence of a magnetic field normal to this surface

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It is shown that the magnetic field normal to the surface of liquid helium has an appreciable effect on the location of the Grimes-Adams resonances in the electron-ripplon system, which occur as a result of Wigner crystallization in this system.

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Grimes and Adams[1] observed a resonance excitation of the electron-ripplon system on the surface of liquid helium in a variable electric field $E_{\parallel}$ parallel to the surface of the helium in a certain temperature range $T < T_c$. The temperature T_c measured in Ref. 1 is related to the average density n_s of the electrons above the helium surface by the relation $\pi^{1/2}e^2n_s^{1/2}/T_c = 137 \pm 15$. The mentioned experiments, whose interpretation is given by Fisher, Halperin, and Platzman,[2] indicate that a phase transition, called the Wigner crystallization, occurs in a two-dimensional electronic system above the helium. The connection between n_s and T_c established in the experiments,[1] which has a clear physical meaning (there is a competition between the temperature and the Coulomb interaction which stimulates the Wigner crystallization in the electronic system) was confirmed by appropriate theoretical calculations.[3,4] Thus, the Wigner electronic crystal turned out to be a real object which exists, at any rate, on the charged surface of helium.

The goal of this paper is to study the Grimes-Adams resonances in the presence of a magnetic field normal to the surface of the helium. For simplicity, we examine below a uniform limiting case in which the longwave oscillations of the electron density along the surface of the helium are missing.

The sought-for dispersion law of the electron-ripplon system under uniform conditions in the presence of a magnetic field normal to the surface of the helium and neglecting the dissipation has the following form:

$$\omega^2 \pm \omega\omega_H - \frac{e^2E_\perp^2}{2am}\sum_{\mathbf{G}}\frac{n_{\mathbf{G}}\,\omega^2}{\omega^2-\omega_{\mathbf{G}}^2} = 0, \tag{1}$$

$$\omega_{\mathbf{G}}^2 = \frac{\alpha}{\rho}|\mathbf{G}|^3, \quad n_{\mathbf{G}} = s^{-1}\exp\left(-\frac{1}{4}\mathbf{G}^2 < \mathbf{u}^2 >\right), \quad \omega_H = \frac{eH}{mc}$$

$$\mathbf{G}_l^2 = l\mathbf{G}_1^2, \quad \mathbf{G}_1^2 = 8\pi n_s/3^{1/2}, \; l = 1, 3, 4, 7, 9... \tag{1a}$$

$$(\eta^2/<\mathbf{u}^2>) << 1, \quad \mathbf{G}\eta << 1.$$

Here ω is the oscillation frequency, $E_\perp$ is the intensity of the applied electric field, ρ and α is the volume density and the surface tension coefficient of helium, $\mathbf{G}$ are the vectors of the reciprocal lattice of the electronic Wigner crystal, $n_{\mathbf{G}}$ are the Fourier components of the electron density of this crystal along the $\mathbf{G}$ vectors, s is the area of the unit cell of the crystal, $\vec{\eta}$ is the oscillation amplitude of the electron spots near the equilibrium states, H is the intensity of the external magnetic field, m is the mass of the free electron, and $\langle \mathbf{u}^2 \rangle$ is the rms displacement of the electron. The inequalities (1a) are necessary conditions for linearization of the original system of equations of motion and for obtaining the dispersion law (1). The second inequality may be violated in the region of large $\mathbf{G}$. In this region, however, the values of $n_{\mathbf{G}}$ are exponentially small, and the corresponding resonances are difficult to observe.

The magnetic field H, besides having a definite influence on the dispersion law (1) via the cyclotron frequency ω_H, is included additionally in the definition of (1) because of the dependence of $\langle \mathbf{u}^2 \rangle$ on H (the fluctuational rms displacement of the electron from the equilibrium state decreases monotonically with increasing magnetic field H).[5]

In the limiting case $H \to 0$ the dispersion law (1) changes to a corresponding equation for determining the natural frequencies from Ref. 2. In this case the natural

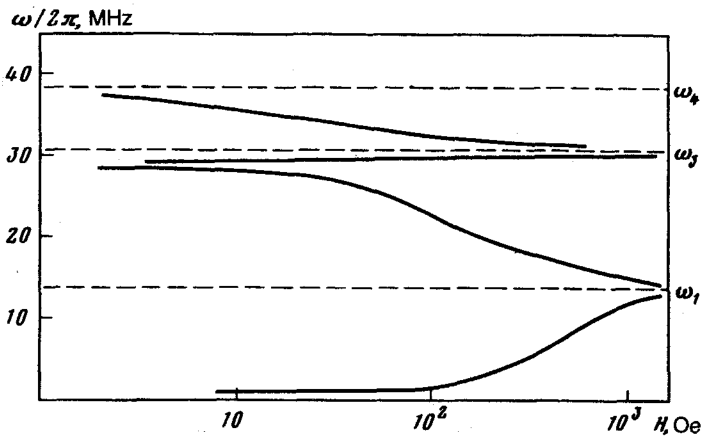

FIG. 1.

frequencies, especially the first frequencies in the ascending order of **G**, are noticeably shifted relative to $\omega_{\mathbf{G}}$. Thus, the minimum natural frequency ω_{min} as $H \to 0$ generally vanishes (see Fig. 1).

In the opposite limiting case $\omega_H \gg \omega_{\mathbf{G}}$ (for the first natural frequency this inequality is slightly more complicated: $\omega_H \gg e^2 E_\perp^2 n_{\mathbf{G}_1}/am \gg \omega_{\mathbf{G}_1}$) the roots of Eq. (1) tend to $\omega_{\mathbf{G}}$

$$\omega \approx \omega_{\mathbf{G}} \pm \Delta\omega, \qquad \Delta\omega = \frac{3e^2 E_\perp^2 n_{\mathbf{G}}}{2\,am\omega_H}, \qquad \Delta\omega \ll \omega_{\mathbf{G}}. \tag{2}$$

Thus, the magnetic field has an effect on the position of the electron-ripplon Grimes-Adams resonances. This may prove to be useful in further experimental study of electron-ripplon resonances.

The characteristic magnetic field intensity, which is necessary for fulfillment of the inequalities $\omega_H \gg \omega_{\mathbf{G}}$, can easily be estimated if the information on the values of $\omega_{\mathbf{G}}$ is available. Thus, for conditions discussed in Ref. 2, i.e., for the electrons on the surface of He^4 at an average density $n_s \approx 4.55 \times 10^8$ cm^{-2}, the frequency $\omega_{\mathbf{G}} = 8.5 \times 10^7$ sec^{-1}. Therefore, the magnetic field $H \approx 10^3$ Oe, for which $\omega_H \approx 10^{10}$ sec^{-1}, is sufficiently strong in this case.

The schematic dependence of the first Grimes-Adams resonances on the magnetic field for the conditions $n_s = 4.55 \times 10^8$ cm^{-2} and $E_\perp \approx 415$ V/cm is shown in Fig. 1. The frequencies ω_l in Fig. 1 have the meaning of $\omega_{\mathbf{G}_l}$.

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