Reluctance of two-dimensional systems

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Superconducting fluctuations contribute noticeably to the reluctance of thin films even at a much higher temperature than the critical and even in the case of repulsion between the electrons. This contribution partially compensates for the contribution to the reluctance due to electron localization, which improves the agreement between the theory and experiment.

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The ordinary reluctance of thin films is small because of the small mean free path of electrons. There exist, however, two quantum effects which lead to a nontrivial dependence of the reluctance on the magnetic field in the region of weak fields. One of these effects, which occurs in a system of noninteracting electrons, is associated with the Anderson localization and is examined in Refs. 1 and 2. Below we examine another effect produced by scattering of electrons by superconducting fluctuations. These fluc-

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tuations are more noticeable near the superconducting transition temperature, but even far from it they lead to a slow, logarithmic temperature dependence of the reluctance. Such dependence is hard to observe; however, it is important that the magnetic field has a strong effect on the interaction of electrons with the superconducting fluctuations. The effect produced as a result of this partially compensates for the localization effect in the system of noninteracting electrons. This conceivably can explain the fact that the reluctance in the experiment[3] is smaller than that predicted by the theory of noninteracting electrons.[1,2] We note that the superconducting fluctuations occur even when the electrons repel each other and the superconducting transition is missing.

At temperatures not too close to the superconducting transition temperature and in a weak magnetic field, the contribution of the fluctuations to the conductivity, which is described by the Maki-Thompson diagram[4] (see Fig. 1), is

$$\omega\sigma_1 = 2e^2 \langle v_x^2 \rangle T^2 \sum_{0<\epsilon,\,\epsilon_1<\omega} \int d\xi\, G(\epsilon)\, G(\epsilon_1)\, G(\omega-\epsilon)\, G(\omega-\epsilon_1)$$
$$\times \frac{eH}{\pi\hbar c} \sum_j C_j(\epsilon+\epsilon_1) C_j(2\omega-\epsilon-\epsilon_1)\, \Gamma\, \frac{|\epsilon-\epsilon_1|}{2\pi T}. \tag{1}$$

Here $\epsilon = \pi T(2n+1)$. The one-particle Green's functions represented in Fig. 1 by the solid lines are

$$G(\epsilon) = (\xi - i\epsilon - i\,\mathrm{sign}\,\epsilon/2\tau)^{-1}.$$

The dashed lines represent the scattering of electrons by impurities. The set of ladder diagrams comprised of solid and dashed lines is given by

$$\tau C_j(\omega) = [\omega + a(j+\tfrac{1}{2}) + 1/\tau_\epsilon]^{-1}, \qquad a = \frac{4DeH}{\hbar c}, \tag{2}$$

where τ_ϵ is the energy relaxation time and $a(j+1/2)$ are the eigenvalues of the $D(\nabla - 2eiA/\hbar c)^2$ operator. The effective amplitude of interaction between the electrons, represented by a wavy line in Fig. 1, is

$$\Gamma(m) = -g[1 + gT\sum_{\epsilon>0} 4\pi\tau C(2\epsilon + 2\pi|m|)]^{-1}. \tag{3}$$

Of interest is the region of weak magnetic fields $a \sim 1/\tau_\epsilon \ll T - T_c$. In this case the dependence of $\Gamma(m)$ on the magnetic field can be disregarded. Thus, it is convenient to express $\Gamma(m)$ in terms of the temperature-dependent interaction constant

$$\Gamma(m) = -[\psi(m+\tfrac{1}{2}) - \psi(\tfrac{1}{2}) + 1/g(T)]^{-1}, \tag{4}$$

where

$$g(T) \equiv -\Gamma(0) = \left(g^{-1} + \ln\frac{\omega_D}{T}\right)^{-1} = -1/\ln T/T_c. \tag{5}$$

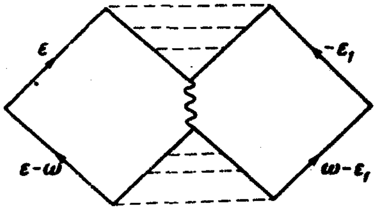

FIG. 1.

The g constant for the electron interaction at the ordering energy is ordinarily expressed in terms of the g_p constant, which is related to the phonon exchange, and in terms of the g_c constant of the Coulomb interaction

$$g = -g_p + g_c/(1 + g_c \ln \epsilon_F/\omega_D) .$$

To calculate the conductivity σ_1, we must calculate expression (1) for $\omega = 2\pi n = 2\pi\Omega$ and then analytically continue it to the region $\mathrm{Re}\Omega > 0$. The result of such continuation has the form

$$\sigma_1 = \frac{2DeH}{\pi^2\hbar c} \sum_j r\, C_j(\omega)\, \beta(T),$$

$$\beta(T) = \frac{1}{\Omega} \sum_{n\geqslant 0,m} \left\{ \Gamma(|m|) \left(\frac{1}{2n+m+1} - \frac{1}{2n-m+1+2\Omega} \right) \right. \tag{6}$$

$$+ \frac{\mathrm{sign}\, m\, \Gamma(|m|+\Omega)}{2n+m+\Omega+1} + \Gamma(2n+1+2\Omega) \tag{7}$$

$$\left. \times \left(\frac{1}{2n-m+1+2\Omega} - \frac{1}{2n-m+1+\Omega} \right) \right\}.$$

In the low-frequency limit we obtain

$$\beta(T) = \frac{\pi^2}{4} \sum_m (-1)^m \Gamma(|m|) - \sum_{n\geqslant 0} \Gamma''(2n+1) . \tag{8}$$

TABLE I.

$-g(T)$	$-0{,}1$	0,1	0,2	0,3	0,5	0,8	1	2	5	10
$\beta(T)$	0,017	0,015	0,06	0,13	0,33	0,73	1,05	3	9,8	22

Substituting $\Gamma(m)$ in this expression from Eq. (4), we determine the dependence of β on $-g = (\ln T/T_c)^{-1}$. This dependence is given in Table I. In the limiting case $T \to T_c$, $-g(T) \gg 1$ and $\beta = -g(T)\pi^2/4$, which coincides with the result obtained by Thompson.[4] In the opposite limiting case of high temperatures $T \gg T_c$, $|g| \ll 1$, $\beta = g^2(T)\pi^2/6$, the parameter β is inversely proportional to the square of the logarithm of the temperature.[5]

The fluctuation conductivity determined by Eq. (6) depends on the magnetic field, like the conductivity in the system of noninteracting electrons,[1,2] but it has an opposite sign. Summing both effects, we obtain the following expression for the reluctance of the square of the film

$$\Delta R = -R^2 \frac{e^2}{2\pi^2\hbar} [\alpha - \beta(T)] \, Y\left(\frac{4DeH\tau_\epsilon}{\hbar c}\right), \tag{9}$$

where

$$Y(x) = \ln x + \psi\left(\frac{1}{2} + \frac{1}{x}\right) = \begin{cases} \dfrac{x^2}{24} & (x << 1) \\ \ln x & (x >> 1) \end{cases}.$$

If the spin-orbit interaction of the electrons with the impurities is small, then $\alpha = 1$. In the opposite limiting case $(\tau_{s0} \ll \tau_\epsilon)$ $\alpha = -1/2$. We note that the parameter β does not depend on the spin-orbit interaction. A large concentration of magnetic impurities suppresses both effects.

The experimental results[3] correspond approximately to the parameter $\alpha - \beta = \alpha n_v = 0.6$. This result can conceivably mean that the interaction between the electrons is relatively strong $|g| \sim 0.5$ in MOSFET films.

A detailed experimental study of the examined effect will make it possible to determine the parameters of the conductor, which are difficult to determine by another method. The dependence of the reluctance on the magnetic field gives the temperature dependence and the time of the energy relaxation τ_ϵ. The coefficient in Eq. (9) is a universal constant $(e^2/2\pi^2\hbar = 1.2\times 10^{-5}\Omega^{-1})$. Therefore, knowing the reluctance we can determine the coefficient β and its temperature dependence, which in turn gives the effective interaction between the electrons $g(T)$. In the case of repulsion, $g(T)$ and hence $\beta(T)$ tend logarithmically to zero with decreasing temperature. In the case of attraction, $\beta(T)$ increases with decreasing temperature. Both effects should be observed in any two-dimensional conducting systems such as MOS (metal-oxide-semiconductor), metallic films, and layered conductors.

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