# Atom motion in a superstrong magnetic field

M. B. Kadomtsev
*M. V. Lomonosov Moscow State University*

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A mass anisotropy effect arises when an atom moves in a superstrong magnetic field $B > 10^{12}$ G. It is shown that a particle with an anisotropic mass performs a complicated motion in an inhomogeneous field: in a forward-current field slowing of the radial velocity leads to elastic scattering, and in a dipole field the sign of the derivative of the centrifugal potential changes, which leads to falling of the particle on the dipole.

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After the discovery of colossal magnetic fields in pulsars ($\sim 10^{12}$ Oe), the behavior of matter in superstrong fields acquired urgency.[1-3] An atom, placed in such a field, is stretched out along the field, while simultaneously being squeezed on all sides. Consequently, the binding energy increases, which leads to the appearance of an attraction into the region of higher field. Besides this, however, as is known, an atom in a sufficiently strong magnetic field becomes more massive in the cross-field direction. It is easy to understand this effect by considering that when an atom moves with a velocity $v_\perp$ at right angles to the field, its electron shells are distorted in the same manner as if an electric field $v_\perp B/c$ were acting. The energy of the polarized atom, on the other hand, is equal to $E = d^2/2\alpha$, where $d = \alpha E$ is the dipole moment and $\alpha$ is the polarization coefficient. Thus, an additional energy $E = \frac{1}{2}(\alpha B^2/c^2)\, v_\perp^2$ appears. The total energy is

$$E = \frac{1}{2} m_0 v^2 + \frac{1}{2} \frac{\alpha B^2}{c^2} v_\perp^2 - U(B). \tag{1}$$

Here $m_0$ is the atomic mass and $U(B)$ is the binding energy.

At large $B$ the last term $U(B)$ is only very slightly dependent on $B$ (as $\ln^2 B$) and, for simplicity, it can be ignored. Let us convert to units in which $m_0 = 1$ and $B$ is measured in units of $c\sqrt{m_0}/\sqrt{\alpha}$; thus, the Lagrangian (1) can be simplified:

$$\mathcal{L} = \frac{1}{2} v_\parallel^2 + \frac{1}{2}(1 + B^2)\, v_\perp^2, \tag{2}$$

where $v_\parallel$, $v_\perp$ are the velocity components along and at right angles to the field, respectively. Let us now consider two specific problems.

a) An atom in the field of a forward current.

In this case $B$ has only azimuthal component (we are using a cylindrical system), and the units of length can be chosen such that $B = R^{-1}$. Such a field, of course, cannot exist under the conditions in outer space; however, this situation can be realized for excitons in a solid. Let us assume that $z = 0$ (motion along $z$ is obvious); thus,

  

$$\mathcal{L} = \frac{1}{2} v_\phi^2 + \frac{1}{2}(1 + R^{-2}) v_R^2 , \tag{3}$$

where $v_\phi = R\dot\phi$; and $v_R = \dot R$. We have the equations of motion: $R^2\dot\phi = M = \text{const}$,

$$(1 + R^{-2})\frac{d^2R}{dt^2} - \frac{1}{R^3}\left(\frac{dR}{dt}\right)^2 - \frac{M^2}{R^3} = 0. \tag{4}$$

The energy conservation law requires that (it can be obtained directly from the equation of motion)

$$\frac{1}{2}(1 + R^{-2})\left(\frac{dR}{dt}\right)^2 + \frac{1}{2}\frac{M^2}{R^2} = \text{const} = v^2/2, \tag{5}$$

where $v$ is the particle velocity at $R = \infty$.

As seen from the preceding expression, the magnetic field slows down the particle's radial velocity (this effect plays a special role at $R < 1$). Let us now determine the minimum approach distance from the condition $dR/dt = 0$. It turns out to be equal to the impact parameter $R_0 = M/v$. But, since the angular velocity $\dot\phi = M/R^2$ retains its former value, the atom (before leaving the region $R < 1$) completes many revolutions. It can be said that the particle sticks for some time in the superstrong field region and is ejected in a direction having arbitrary $\phi$.

b) An atom in a dipole field.

In the $R$, $\theta$, $\phi$ spherical coordinate system the equation for a field line is given by

$$R = L \sin^2\theta, \tag{6}$$

where $L = \text{const}$. Let us choose units in such a way that the field $B = 1$ for $L = 1$, $\sin\theta = 1$. Thus,

$$\mathbf{B} = \frac{R^2\mathbf{e}_z - 3\mathbf{R}(\mathbf{e}_z\mathbf{R})}{R^5}; \quad B^2 = \frac{1 + 3\cos^2\theta}{R^6} . \tag{7}$$

Let us use the quantity $L = R/\sin^2\theta$ as one of the generalized coordinates. As the second (orthogonal to $L$) coordinate we choose $s = \cos\theta/R^2$, and we shall retain $\phi$ as the third. Then the Lagrangian assumes the form

$$\mathcal{L} = \frac{1}{2} g_{LL}(1 + B^2)\dot L^2 + \frac{1}{2} g_{ss}\dot s^2 + \frac{1}{2}(1 + B^2)\rho^2\dot\phi^2 . \tag{8}$$

Here $g_{LL}$, $g_{ss}$ are components of the metric tensor, and $\rho = R\sin\theta$ is the distance to the symmetry axis. Let us assume that the motion is localized in the $s = 0$ plane, i.e., $\theta = \pi/2$. Thus, Eq. (8) can be simplified:

$$L = \frac{1}{2}(1 + L^{-6})\dot L^2 + \frac{1}{2}(1 + L^{-6})\dot\phi^2 L^2. \tag{9}$$

We can introduce the Hamiltonian

$$H = \frac{1}{2}\frac{1}{1+L^{-6}}K^2 + \frac{1}{2}\frac{M^2}{L^2(1+L^{-6})}, \tag{10}$$

where $K$ is the momentum conjugate to $L$ and $M$ is the angular momentum. The energy conservation law (which again can be obtained from the equation of motion) can be written as follows:

$$(1+L^{-6})\dot{L}^2 + \frac{M^2}{L^2(1+L^{-6})} = v^2 = \text{const}. \tag{11}$$

Let us find the reversal point, where $\dot{L} = 0$. For it we obtain the equation

$$L_0^2\, L^4/(L^6+1) = 1, \tag{12}$$

where $L_0 = M/v$ is the impact parameter. It is easy to see that this equation has real solutions only for $L_0 > (3)^{1/2}/(2)^{1/3} = 1.37$. Two roots are obtained in this case: the larger root corresponds to the reversal point of a particle coming from $\infty$, the smaller one corresponds to that coming from the center.

For $L_0 < (3)^{1/2}/(2)^{1/3}$, $\dot{L}$ does not become 0—this is due to the fact that for $L < 1$ the centrifugal potential does not increase, but decreases with decreasing $L$. For $L \ll 1$ Eq. (11) becomes (approximately):

$$L^{-6}\dot{L}^2 = v^2 - L_0^2 v^2 L^4. \tag{13}$$

The second term is small (for $L_0 < 1$). Ignoring it, we obtain the approximate solution $L = 1/[2v(y-t_0)]^{1/2}$ for a particle arriving from $L = \infty$.

Let us now consider arbitrary motion. The Hamiltonian is written in the form

$$H = \frac{1}{2}\frac{1}{g_{LL}(1+B^2)}K^2 + \frac{1}{2g_{ss}}P^2 + \frac{M^2}{2(1+B^2)\,L^2\sin^6\theta}. \tag{14}$$

Here $K$, $P$, and $M$ are the generalized momenta, corresponding to the coordinates $L$, $s$, $\phi$. It is easy to show that

$$B^2 = \frac{1+3\cos^2\theta}{L^6\sin^2\theta}; \quad g_{LL} = \frac{\sin^6\theta}{1+3\cos^2\theta}; \quad g_{ss} = \frac{\cos^3\theta}{s^3(1+3\cos^2\theta)};$$

the last term in the Hamiltonian has a maximum at $\theta = \pi/2$. Accordingly, with respect to motion along a force line the point $\theta = \pi/2$ corresponds to a "hump" on the potential, and near $\theta = \pi/2$ the motion with respect to $\theta$ is unstable. Thus, for $B > 1$ the atom falls along a force line to the field source, which is again attributable to the fact that the centrifugal potential decreases, rather than increases, as we approach the center.

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