Instability of quasi-one-dimensional electron chain and the "string-zigzag" structural transition

A. V. Chaplik
Institute of Semiconductor Physics, USSR Academy of Sciences, Siberian Branch

(Submitted 17 January 1980)
Pis'ma Zh. Eksp. Teor. Fiz. **31**, No. 5, 275–278 (5 March 1980)

The linear chain of electrons, held on the surface of liquid helium by the field of a charged filament, is unstable with respect to lateral displacements if the density of the system is above some critical value. It is shown that in this case the energy minimum is realized with a zigzag-shaped arrangement of the particles. Estimates show that such structural transition can be detected by modern experimental techniques.

PACS numbers: 73.20.Cw

Recent progress in experiments with two-dimensional electrons on the surface of liquid helium[1,2] show that one can work with a system in which the total number of electrons is extremely small ($\sim 10^6$). This makes it possible to raise the question of studying one-dimensional structures with a helium cell having completely reasonable dimensions of ~ 1–10 cm.

This paper investigates the vibrational spectrum of a quasi-one-dimensional chain of electrons. It is shown that for a certain relationship among the parameters of the system the arrangement of electrons "in a line" is unstable with respect to transverse vibrations of the chain. This leads to the "string-zigzag" structural transition.

Let us assume that in an ordinary system containing a two-dimensional electron gas, a filament, to which a positive potential is applied, is located at a depth H beneath the surface of the helium. The filament radius is $r_0 \ll H$, and let us denote the positive charge density of the wire by κ. A MIS [metal-insulator-semiconductor] structure with a narrow field electrode is another example; in this case it is the insulator thickness.[4] Let us assume that the direction of the wire is the "y" axis, and the perpendicular direction to the wire on the surface plane is the x axis. The charged wire produces a potential "groove" for electrons $v(x) = 2e\kappa \ln(H^2 + x^2)^{1/2}/H$ on the He surface. Taking into account the Coulomb repulsion of the particles, we obtain an expression for the total potential energy U of the system that is valid for small deviations x_i, y_i from the equilibrium positions in the linear chain:

$$U = \frac{e^2}{2a^3} \sum_{i \neq j} \frac{(y_i - y_j)^2}{|i-j|^3} - \frac{e^2}{4a^3} \sum_{i \neq j} \frac{(x_i - x_j)^2}{|i-j|^3} + \frac{e\kappa}{H^2} \sum_i x_i^2 . \qquad (1)$$

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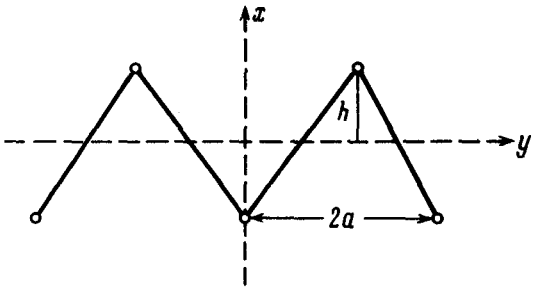

FIG. 1.

Here, α is the period of a one-dimensional chain, which is determined by the linear electron density. At finite temperatures, of course, long-range order is impossible in a one-dimensional system. As estimates show (see below), however, in the chain there can be rather long crystalline portions at temperatures of a few hundred degrees Kelvin. The solution of the vibrational problem with the potential (1) gives the following frequency spectrum (q is the "phonon" wave vector): longitudinal branch (vibrations along the chain)

$$\omega_{\parallel}^2 = \frac{4e^2}{ma^3} \sum_{n=1}^{\infty} \frac{1 - \cos qan}{n^3}, \tag{2a}$$

transverse branch (vibrations are perpendicular to the chain)

$$\omega_{\perp}^2 = \frac{2e\kappa}{mH^2} - \frac{2e^2}{ma^3} \sum_{n=1}^{\infty} \frac{1 - \cos qan}{n^3}. \tag{2b}$$

The longitudinal branch for $qa \ll 1$ (the continuous-medium approximation) corresponds to one-dimensional plasma waves $\omega_{\parallel}^2 \approx (2e^2q^2/ma)\ln(1/qa)$.[3] The transverse branch is described by a dispersion law like that of optical phonons: its frequency decreases as q changes from 0 to π/a. If the inequality

$$\frac{\kappa}{H^2} < \frac{7}{4} \zeta(3) \frac{e}{a^3}, \tag{3}$$

is satisfied [$\zeta(x)$ is the Riemann function], then $\omega_{\perp}^2 < 0$ for $q = \pi/a$. Thus, the system is unstable with respect to transverse displacements: the repulsion of the "rest" of the electrons overcomes the attraction of each of them to the wire.

Let us now show that the zigzag structure (see Fig. 1) ensures an energy minimum at some value of h—the height of the link break—if criterion (3) is satisfied, i.e., the linear arrangement is unstable. Let us calculate the difference in the potential energies (per electron) for the zigzag and linear structures. This difference is finite, whereas the energy of each structure diverges logarithmically with an increase in the chain length. We have

$$\frac{\Delta U}{N} = e\kappa \ln\left(1 + \frac{h^2}{H^2}\right) + \frac{2e^2}{a}\sum_{n=1}^{\infty} \times \left[\frac{1}{\sqrt{4h^2/a^2 + (2n-1)^2}} - \frac{1}{2n-1}\right]. \quad (4)$$

An analysis of (4) shows that when (3) is satisfied, $\Delta U \sim (-h^2) < 0$ for $h \ll a$, while for $h \gg a$

$$\frac{\Delta U}{N} \approx e\kappa \ln(1 + h^2/H^2) - \frac{e^2}{a} \ln h/a . \quad (5)$$

By having three independent parameters—κ, a, and H—one can always achieve a positive ΔU for large h (as well as when the electrical neutrality condition $\kappa = e/a$ is satisfied). Thus, there exists an h_0 that gives an energy minimum for the zigzag structure. At sufficiently small h, of course, there is no minimum, i.e., absolute system instability sets in.

For a wire radius $r_0 \sim 10^{-4}$ cm it is easy to obtain a charge density of $\sim 10^7$ $e\cdot\text{cm}^{-1}$ on it. If the distance to the surface is $H \sim 10^{-3}$ cm, then the critical electron density, at which the structural transition occurs, is $a_0^{-1} \sim 2\times 10^4$ cm^{-1}. A system of 10 parallel wires, each 5 cm long, gives $\sim 10^6$ total number of electrons, which, as already mentioned, is sufficient to detect the effect. The detection method, obviously, can be the same as in Ref. 1, since the location of riplon resonances is strongly dependent on the structure symmetry. Another possibility is that of observing a puncturing of the helium surface: with a continuous decrease of κ the width of the strip, filled with electrons, should increase at $\kappa < \kappa_c$ (abruptly if $\partial^4 U/\partial h^4 \sim 93\zeta(5)eH^4 - 4\kappa a^5 < 0$).

Because of the linear divergence at $T > 0$ of the rms displacement $\langle u^2 \rangle$, only the restricted portions of it may be ordered for an infinite chain. The dimension of such ordered portion is $4\pi^2 a(mv^2/T) \sim (8\pi^2 e^2/T)\ln 4\pi e^2/a$, where v is the velocity of the plasma wave. The longitudinal branch destroys the long-range order, whereas the slot in the spectrum of the transverse branch ensures the finiteness of its contribution to $\langle u^2 \rangle$. For $\alpha \sim 2\times 10^{-4}$ cm we obtain a length $L_{\max} \sim 5\times 10^3\, \alpha/T$ for the "crystalline" portion, i.e., for $T \sim 0.05$ K $L_{\max} \sim 10^5\, \alpha = 20$ cm.

In conclusion, the author wishes to thank V. L. Pokrovskiĭ for helpful advice, and Yu. V. Monarkh and A. S. Rybalko for discussing the work.

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