Phase transition in a planar magnetic material with frustrations

V. S. Dotsenko, Jr.

(Submitted 24 January 1980)
Pis'ma Zh. Eksp. Teor. Fiz. **31**, No. 5, 301–304 (5 March 1980)

The dependence of the temperature of the Berezinskiĭ transition on the density of antiferromagnetic bonds n_{af} is obtained for a planar magnetic material (xy-model). It is shown that this density plays the role of an effective temperature, with the phase transition disappearing above a certain density n^*_{af}.

PACS numbers: 75.40.Bw

In this paper we examine a planar magnetic material (xy-model), in which a small number of antiferromagnetic bonds are introduced in a random manner. The appearance of these bonds leads to the formation of so-called frustrations,[1,2] which are vortex-type topological formations with a half charge.[2] If the density $n_{\alpha f}$ of antiferromagnetic bonds is small, then all frustrations are paired in such a manner that the topological charge of the pair is equal to zero. Such pair of frustrations constitutes a dipole which is formed by charges with a modulus equal to one-half the vortex-charge $p = \sqrt{2\pi J}$. J is the coupling constant in energy:

$$H = J \int d^2 r \, (\vec{\nabla}\phi)^2 \tag{1}$$

ϕ is the rotation angle of the spins. The size of these dipoles is equal, in order of magnitude, to the lattice dimension. At temperatures different from zero a dipole, held in the lattice, has a degree of freedom which is attributable to the fact that the dipole can alter the sign of its direction by flipping of a vortex from one frustration to the other. Because of the long-range interaction of dipoles (interaction energy $\sim r^{-2}$), the possibility of a phase transition to a state, in which the direction of each dipole is frozen, is not excluded in principle. There are indications,[3] however, that such phase transition is missing in our system (the power of the potential decrease is the same as the dimensionality of the space), and everything that follows will be done with this assumption.

As Berezinskiĭ showed,[4] a "pure" planar magnet has a phase transition which is attributable to the fact that above a certain temperature the vortex pairs start to dissociate.[5] In our case, in addition to the ordinary vortex pairs whose size and number are controlled by the temperature, there are also forcibly introduced half-vortices. Thus, the energy of a system with m pairs of temperature vortices, ignoring spin waves, can be written in the form[5]:

$$H = -\sum_{i \neq j} q_i q_j \ln \left| \frac{\mathbf{r}_i - \mathbf{r}_j}{\mathbf{r}_0} \right| + 2\mu m, \qquad |\mathbf{r}_i - \mathbf{r}_j| \geqslant r_0 \,. \tag{2}$$

The quantity 2μ is the energy necessary for the formation of one pair of temperature vortices. The summation is done over all vortices and frustrations, and for the frustrations it must be assumed that $|q| = \frac{1}{2}p$. In addition to integration over all possible positions of the moving vortices, in the statistical sum we must now also take into account all possible distributions of the frustration signs with allowance for the overall neutrality:

$$Z = \sum_{\text{over frustration signs}} \sum_{m=0}^{\infty} \frac{K^{2m}}{(m!)^2} \left(\prod_{j=1}^{2m} \int_{D_j} d^2 r_j \right) \exp\left\{ \beta \sum_{k \neq l} q_k q_l \ln \left| \frac{\mathbf{r}_k - \mathbf{r}_l}{r_0} \right| \right\}. \tag{3}$$

Here $r_0^2 K = \exp(-\beta\mu)$, and the integration domain D_j is defined by the condition that the vortices cannot be closer than the lattice dimension r_0. If, following Ref. 6, we now perform a renormalization in (3) by changing the scale $r_0 \rightarrow r_0 + dr_0$, then we obtain the same equations as in Ref. 6:

$$\frac{d(Kr_0^2)}{dr_0} = -(\beta p^2 - 2)\frac{1}{r_0},$$

$$\frac{d(\beta p^2)}{dr_0} = -2\pi^2 \beta p^2 (Kr_0^2)^2 \frac{1}{r_0}, \tag{4}$$

which leads to the existence of the same phase transition as in the "pure" model. The transition temperature is determined by the condition:

$$\frac{1}{T_c} p_1^2 \approx 2 \tag{5}$$

[ignoring the small exponential quantity $\exp(-\beta_c\mu)$], except that now the vortex interaction constant p_1^2 depends on the temperature because of the polarizability of the dipoles. It is easy to obtain this polarizability. The energy of the dipole system in an external field $\mathbf{E}$ has the form

$$U = -|\mathbf{E}| p r_0 \sum_i \sigma_i \cos\theta_i - \sum_{i \neq j} \kappa_{ij} \sigma_i \sigma_j. \tag{6}$$

Here θ_i are the fixed dipole rotation angles $\sigma = \pm 1$, and the summation is done over all dipoles. The average dipole moment $\langle \bar{D} \rangle$ is directed along the field $\mathbf{E}$:

$$\overline{\langle D \rangle} = \left(\prod_i \frac{1}{\pi} \int_0^{\pi} d\theta_i \right) \frac{\sum_{\{\sigma\}} \left(\sum_i \tfrac{1}{2} p r_0 \sigma_i \cos\theta_i \exp\{-\beta U\} \right)}{\sum_{\{\sigma\}} \exp\{-\beta U\}}.$$

For the polarizability $\frac{\partial}{\partial E}\langle \bar{D} \rangle|_{E=0}$ we obtain:

$$\left(\prod_i \frac{1}{\pi} \int_0^{\pi} d\theta_i \right) \tfrac{1}{2} \beta p^2 r_0^2 \sum_{i,j} \langle \sigma_i \sigma_j \rangle \cos\theta_i \cos\theta_j.$$

Taking into account that $\langle\sigma_i\sigma_j\rangle = \langle\sigma^2\rangle\delta_{ij} = \delta_{ij}$ (since the entropy of a unit area is not equal to zero even at $T = 0$) and also that the dipole density is equal to $2n_{af}$, we obtain the dielectric constant of the medium:

$$\epsilon = 1 + 2\beta\pi^2 J n_{af}. \tag{7}$$

Therefore, the statistical weight of a pair of vortices has the form

$$-\frac{4\pi J}{T + 2\pi^2 J n_{af}} \ln\frac{r}{r_0} - \frac{2\mu}{T}. \tag{8}$$

From (5) we obtain the equation for the phase-transition curve

$$T_c + 2\pi^2 J n_{af} = \pi J. \tag{9}$$

For $n_{af} n^*_{af} = 1/2\pi$ the transition temperature vanishes. The expression

$$T_{\text{eff}} = T + 2\pi^2 J n_{af} \tag{10}$$

plays the role of the effective temperature of the xy-model with a subsystem of frustrations, since the spin–spin correlator is defined by this expression

$$\overline{\langle(\mathbf{s}(\mathbf{r}), \mathbf{s}(\mathbf{r}'))\rangle} = \left|\frac{\mathbf{r} - \mathbf{r}'}{r_0}\right|^{-\frac{T_{\text{eff}}}{4\pi J}}. \tag{11}$$

In fact, the spin-deviation angle ϕ can be represented in the form $\phi = \psi + \bar{\phi}$, where ψ are small temperature deviations, and $\bar{\phi}$ is determined by the configuration of the dipoles:

$$\bar{\phi}(\mathbf{r}) = \tfrac{1}{2}\sum_i \frac{(\mathbf{a}_i, \mathbf{r} - \mathbf{x}_i)}{|\mathbf{r} - \mathbf{x}_i|^2}\sigma_i$$

($\mathbf{a}_1$ is the direction and x_i is the location of the ith dipole). Therefore,

$$\overline{\langle(\mathbf{s}(\mathbf{r}), \mathbf{s}(\mathbf{r}'))\rangle} =$$

$$\left(\prod_i \frac{1}{S}\int d^2x_i \frac{1}{\pi}\int_0^\pi d\theta_i\right) \langle \exp\{i(\bar{\phi}(\mathbf{r}) - \bar{\phi}(\mathbf{r}'))\}\rangle \left|\frac{\mathbf{r} - \mathbf{r}'}{r_0}\right|^{-\frac{T_{\text{eff}}}{4\pi J}}$$

(S is the system area). A calculation leads to the result (11).

Thus, under the curve (9) we have the problem of the "pure" xy-model with an effective temperature (10). It follows from this, in particular, that the critical index of the phase transition is conserved.

We note that the result (11), unlike (8), does not depend on the assumption of no frozen state for the dipole subsystem.

The author wishes to thank G. E. Volovik for suggesting the problem and for useful discussions, and also V. L. Pokrovskiĭ and M. V. Feigel'man for useful discussions.

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