Stark-cyclotron resonance in semiconductors with a superlattice

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The existence of a static-current resonance in semiconductors with a superlattice is predicted theoretically when the multiple Larmor frequency coincides with the multiple Stark frequency.

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At present, a number of resonance effects in semiconductors, in particular, the cyclotron resonance in ordinary semiconductors[1] and the Stark resonance in semiconductors with a superlattice[2] have been thoroughly investigated.

Any type of electron resonance is associated with the motion of electrons in any two fields, each of which leads to a finite motion with a characteristic frequency, one of which plays the role of the natural frequency of the electron and the other has the role of the the frequency of the external force. A coincidence of these frequencies produces a resonance. In the cyclotron resonance the Larmor frequency plays the role of the natural frequency and the frequency of the variable electric field plays the role of the frequency of the external force. In the Stark resonance the natural oscillation frequency of the electron—the Stark frequency and the frequency of the induced field—as usual is the frequency of the electromagnetic field. We note that the electronic oscillations with the Stark frequency in relatively small electric fields occur in semiconductors with a superlattice.

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If the periodicity of the dispersion law is essential as a function of the quasi momentum, then the electron has a finite, periodic motion in a constant electric field with the Stark frequency and in a constant magnetic field with the Larmor frequency. We can expect that the constant current will have a resonance behavior when the Stark frequency coincides with the cyclotron frequency. We call this resonance the Stark-cyclotron resonance. Since the dispersion law for an electron is nonquadratic, the resonance also occurs at multiple Larmor and Stark frequencies.

For simplicity, we shall limit ourselves to a one-dimensional super-lattice for which the dependence of energy on the quasi momentum can be written as follows[3]

$$\epsilon(\mathbf{p}) = \epsilon_{\parallel}(\mathbf{p}_{\parallel}) + \mathbf{p}_{\perp}^2/2m\,, \tag{1}$$

where ϵ is the electron energy, $\mathbf{p}_{\perp}$ is the quasi-momentum component directed perpendicularly to the axis of the superlattice, m is the effective electron mass characterizing the motion of an electron perpendicularly to the axis of the superlattice, $\epsilon_{\parallel}(\mathbf{p}_{\parallel})$ is the part of the energy that characterizes the motion of an electron along the axis of the superlattice, $\mathbf{p}_{\parallel}$ is the quasi-momentum component along the axis of the superlattice, and $\epsilon_{\parallel}(\mathbf{p}_{\parallel})$ is the periodic function of the quasi-momentum $\mathbf{p}_{\parallel}$.

To obtain a Stark-cyclotron resonance, the constant electric field **E** must be directed parallel to the axis of the superlattice and the magnetic field **H** must be directed at an angle to the axis of the superlattice. It can be show that the condition for a resonance in this case looks as follows:

$$n_1\Omega = n_2\omega_{\parallel}\,. \tag{2}$$

Here $\Omega = \frac{1}{\hbar}eEa$ is the Stark frequency, a is the superlattice constant, $\omega_{\parallel} = eH_{\parallel}/mc$ is the Larmor frequency, $H_{\parallel}$ is the magnetic-field component along the axis of the superlattice, and n_1 and n_2 are integers.

The magnetic-field component $H_{\perp}$ relates the finite motion along the axis of the superlattice to the Stark frequency and the finite motion in the perpendicular plane to the axis of the superlattice to the Larmor frequency, and, although $H_{\perp}$ is not a part of the resonance condition the presence of this component is necessary for its existence.

We should note that the constant electric field has a double role. On the one hand, it produces a constant current, and on the other, it forms the resonance frequency. In the cyclotron resonance these functions are divided between the amplitude and frequency of the alternating current.

Let us calculate the current density. We shall proceed from the kinetic equation in the ν approximation

$$\left(e\mathbf{E} + \frac{e}{c}[\mathbf{v},\mathbf{H}]\right)\frac{\partial f}{\partial \mathbf{p}} = -\nu(f - f_0)\,, \tag{3}$$

where $\mathbf{v} = \partial\epsilon/\partial\mathbf{p}$ is the electron velocity, ν is the characteristic frequency of collisions, and f_0 is the equilibrium Boltzmann distribution function. The following relation can be obtained for the electric current

$$\mathbf{j} = \frac{2e}{(2\pi\hbar)^3}\int d\mathbf{p}\, f_0(\mathbf{p})\,\nu\int_0^{\infty} e^{-\nu\tau}\,\mathbf{v}(\mathbf{p}^{\tau})\,d\tau\,, \tag{4}$$

where p^{τ} satisfies the equation

$$\frac{d\mathbf{p}^{\tau}}{d\tau} = e\mathbf{E} + \frac{e}{c}[\mathbf{v}(\mathbf{p}^{\tau}), \mathbf{H}]. \tag{5}$$

As indicated above, the electric field is directed along the axis of the superlattice. For specificity, we use the following expression for $\epsilon_{\parallel}$[3]:

$$\epsilon_{\parallel} = \Delta\epsilon\left(1 - \cos\frac{\mathbf{p}_{\parallel} a}{\hbar}\right). \tag{6}$$

We shall assume, moreover, that $|\Omega|$, $|\omega_{\parallel}| \gg |\omega_{\perp}| = |e|H_{\perp}/mc$. Using the last assumption, we can solve the equation by iterations according to the parameters $H_{\perp}/H_{\parallel} \ll 1$ and $|\omega_{\perp/\Omega} \ll 1$, after which for the electric current we obtain

$$j_{\parallel} = \frac{eN\Delta\epsilon a}{\hbar} e^{-mT\left(\frac{H_{\perp} a}{H_{\parallel}\hbar}\right)^2} \frac{I_1\left(\frac{\Delta\epsilon}{T}\right)^2}{I_0\left(\frac{\Delta\epsilon}{T}\right)} \sum_{n=-\infty}^{\infty} \frac{\nu(\Omega - h\omega_{\parallel})}{\nu^2 + (\Omega - h\omega_{\parallel})^2} I_n\left(mT\left(\frac{H_{\perp} a}{H_{\parallel}\hbar}\right)^2\right). \tag{7}$$

Here N is the electron concentration and I_n is the modified Bessel function.

As seen from Eq. (7), the resonance condition corresponds to condition (2) in which we should assume that $n_1 = 1$. This is attributable to the specific form of the dispersion law (6). The corresponding resonance component reaches a maximum or a minimum at the point that is shifted by $\pm\nu$ relative to the resonance, rather than at the resonance point. In this sense the behavior of the current in the Stark-cyclotron resonance resembles the behavior of the imaginary part of the high-frequency conductivity in the cyclotron resonance.[1] The extremal values of the resonance current are determined by such relation

$$j_{ext} = \pm\frac{eN\Delta\epsilon a}{\hbar} e^{-mT\left(\frac{H_{\perp} a}{H_{\parallel}\hbar}\right)^2} \frac{I_1\left(\frac{\Delta\epsilon}{T}\right)}{I_0\left(\frac{\Delta\epsilon}{T}\right)} I_n\left(mT\left(\frac{H_{\perp} a}{H_{\parallel}\hbar}\right)^2\right). \tag{8}$$

Of course, to obtain a sufficiently sharp resonance, the inequalities $|\Omega|$ and $|\omega_{\parallel}| \gg \nu$ must be fulfilled.

As seen in the relation (8), the current changes its sign, and in the neighborhood of the resonance there is a negative absolute conductivity and, therefore, an instability of the electric current is possible.

If the electric field, along with the constant, has a variable component with respect to time, which changes periodically with the frequency ω, then the resonance condition can be re-written as follows:

$$n_1\Omega = n_2\omega_{\parallel} + n_3\omega, \tag{9}$$

where n_3 is an integer.

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