

Maxwell supermultiplet in the Wheeler–Feynman approach

A. A. Zheltukhin and V. V. Tugaï

Kharkov Physicotechnical Institute, 310108 Kharkov, Ukraine;
Scientific Physicotechnological Center, 310145 Kharkov, Ukraine

(Submitted 14 July 1994)

Pis'ma Zh. Eksp. Teor. Fiz. **60**, No. 5, 305–310 (10 September 1994)

A supersymmetric generalization of Wheeler–Feynman potentials is proposed.

The electrodynamics of a Maxwell supermultiplet constructed from the world coordinates of charged particles in the superspace is analyzed.

© 1994 American Institute of Physics.

In the Wheeler–Feynman theory,¹ the electromagnetic field a_μ describing the interaction of two charges with world coordinates $x^\mu(t)$ and $y^\mu(\tau)$ is constructed from the coordinates themselves and can be written

$$a^\mu(x) = e \int d\tau \dot{y}^\mu(\tau) \delta(s_0^2), \quad (1)$$

where $s_0^\mu \equiv x^\mu - y^\mu(\tau)$ is the world interval, and $\delta(s_0^2)$ is a δ -function on a cone.

A generalization of the principle of “action at a distance” to the superspace $Z^M = (x^\mu, \theta^\alpha, \bar{\theta}_{\dot{\alpha}})$, $\zeta^M = (y^\mu(\tau), \xi^\alpha(\tau), \bar{\xi}_{\dot{\alpha}}(\tau))$, was proposed in Ref. 2. That generalization includes, along with the ordinary world coordinates x_μ and y_μ , their spinor Grassmann superpartners θ^α and ξ^α (Ref. 3). This generalization has made it possible to expand the concept of constructing fields from coordinates to spinor fields and to construct both electromagnetic and spinor fields from the supercoordinates of neutral particles which have an anomalous magnetic moment.

In this letter we solve the problem of a supersymmetric generalization of Wheeler–Feynman potentials.¹ We construct a superfield Maxwell’s electrodynamics^{4,5} using exclusively the world supercoordinates of the charged particles as fundamental generators of the theory.

Following the lines of Refs. 1–5, we introduce the supersymmetric world interval s_μ and the supersymmetric generalized velocities ω_τ^μ , $\dot{\xi}^\alpha(\tau)$, and $\dot{\bar{\xi}}_{\dot{\alpha}}(\tau)$:

$$s^\mu = x^\mu - y^\mu - i(\theta\sigma^\mu\bar{\xi} - \xi\sigma^\mu\bar{\theta}), \quad (2)$$

$$\omega_\tau^\mu = \dot{y}^\mu - i(\dot{\xi}\sigma^\mu\bar{\xi} - \xi\sigma^\mu\dot{\bar{\xi}}). \quad (3)$$

These quantities replace the ordinary interval s_0^μ and the ordinary velocity $\dot{y}^\mu(\tau)$.

To formulate the generalization proposed here, it is convenient to use left and right chiral bases,⁴

$$\begin{aligned}
x_L^\mu &\equiv x^\mu + i\theta\sigma^\mu\bar{\theta}, & y_L^\mu &\equiv y^\mu + i\xi\sigma^\mu\bar{\xi}, \\
x_R^\mu &\equiv x^\mu - i\theta\sigma^\mu\bar{\theta}, & y_R^\mu &\equiv y^\mu - i\xi\sigma^\mu\bar{\xi},
\end{aligned} \tag{4}$$

and corresponding left and right supersymmetric intervals $s_L^M = (s_L^\mu, \Delta^\alpha)$, $s_R^M = (s_R^\mu, \bar{\Delta}^{\dot{\alpha}})$. Their vector and spinor components are given by²

$$\begin{aligned}
s_L^\mu &= x_L^\mu - y_R^\mu - 2i\theta\sigma^\mu\bar{\xi} = s^\mu + i\Delta\sigma^\mu\bar{\Delta}, & \Delta^\alpha &= \theta^\alpha - \xi^\alpha, \\
s_R^\mu &= x_R^\mu - y_L^\mu + 2i\xi\sigma^\mu\bar{\theta} = s^\mu - i\Delta\sigma^\mu\bar{\Delta}, & \bar{\Delta}^{\dot{\alpha}} &= \bar{\theta}^{\dot{\alpha}} - \bar{\xi}^{\dot{\alpha}}.
\end{aligned} \tag{5}$$

Using the generalized velocities and chiral intervals in (5), we can construct invariant spinor Abelian connections $A^\alpha(x_R, \bar{\theta})$, $\bar{A}^{\dot{\alpha}}(x_L, \theta)$:

$$\begin{aligned}
A_\alpha(x_R, \bar{\theta}) &= e \int d\tau (\omega_{\tau\mu}\sigma_{\alpha\dot{\alpha}}^\mu \bar{\Delta}^{\dot{\alpha}} + 2i\dot{\xi}_{-\alpha}\bar{\Delta}) \delta(s_R^2), \\
\bar{A}_{\dot{\alpha}}(x_L, \theta) &= -(A_\alpha)^* = -e \int d\tau (\omega_{\tau\mu}\Delta^\alpha\sigma_{\alpha\dot{\alpha}}^\mu - 2i\dot{\xi}_{\dot{\alpha}}\Delta) \delta(s_L^2).
\end{aligned} \tag{6}$$

These connections satisfy the chiral conditions and thus the constraints⁴

$$D_\alpha A_\beta = 0 = \bar{D}_{\dot{\alpha}} \bar{A}_{\dot{\beta}} \Rightarrow F_{\alpha\beta} = 0 = F_{\dot{\alpha}\dot{\beta}}. \tag{7}$$

The supersymmetric generalization of the Wheeler–Feynman electromagnetic potential¹ which we are seeking is thus found from the solution of the constraint $F_{\alpha\dot{\beta}} = 0$:

$$A_\mu(x, \theta, \bar{\theta}) = -\frac{i}{4} \bar{\sigma}_{-\mu}^{\dot{\alpha}\alpha} (D_\alpha \bar{A}_{\dot{\alpha}} + \bar{D}_{\dot{\alpha}} A_\alpha).$$

Using (6), we can express this generalization in terms of the world coordinates of the superparticles:¹⁾

$$\begin{aligned}
A_\mu(x, \theta, \bar{\theta}) &= -ie \int d\tau \left\{ \omega_{\tau\mu} - \varepsilon_{\mu\nu\rho\lambda} \omega_\tau^\nu (\Delta\sigma^\rho\bar{\Delta}) \partial^\lambda + i[(\Delta\sigma_\mu\bar{\xi}) - (\dot{\xi}\sigma_\mu\bar{\Delta})] \right. \\
&\quad \left. + \frac{1}{4} \Delta\Delta\bar{\Delta}\bar{\Delta} \omega_\tau^\nu (\partial_\mu\partial_\nu - \eta_{\mu\nu}\square) + [\Delta\Delta(\dot{\xi}\bar{\sigma}_{\mu\rho}\bar{\Delta}) + (\dot{\xi}\sigma_{\mu\rho}\Delta)\bar{\Delta}\bar{\Delta}] \partial^\rho \right\} \delta(s^2).
\end{aligned} \tag{8}$$

The choice of representations (6) and (8) for the connection $A_M(x, \theta, \bar{\theta})$ automatically fixes the superfield Lorentz gauge:

$$\partial^\mu A_\mu(x, \theta, \bar{\theta}) = 0. \tag{9}$$

The allowed gauge transformations $A'_\mu = A_\mu + i\partial_\mu\Lambda$, $A'_\alpha = A_\alpha + iD_\alpha\Lambda$, $\bar{A}'_{\dot{\alpha}} = \bar{A}_{\dot{\alpha}} + i\bar{D}_{\dot{\alpha}}\Lambda$ are now characterized by a real scalar superfield $\Lambda(x, \theta, \bar{\theta})$, which is limited by the conditions

$$\square\Lambda = 0, \quad D^\alpha D_\alpha\Lambda = 0, \quad \bar{D}_{\dot{\alpha}}\bar{D}^{\dot{\alpha}}\Lambda = 0. \tag{10}$$

The nonzero stresses $F_{MN}(x, \theta, \bar{\theta})$, which correspond to the potentials $A_M(x, \theta, \bar{\theta})$, are constructed from the chiral superfields W_α and $\bar{W}_{\dot{\alpha}}$:

$$\begin{aligned}
W^\alpha &\equiv \frac{i}{4} F_{\mu\dot{\alpha}} \bar{\sigma}^{\mu\dot{\alpha}\alpha} = \frac{1}{8} \bar{D}_\beta \bar{D}^{\dot{\beta}} A^\alpha + \frac{i}{2} \partial^{\dot{\alpha}\alpha} \bar{A}_{\dot{\alpha}}, \\
\bar{W}^{\dot{\alpha}} &\equiv \frac{i}{4} \bar{\sigma}^{\mu\dot{\alpha}\alpha} F_{\mu\alpha} = -\frac{1}{8} D^\beta D_\beta \bar{A}^{\dot{\alpha}} + \frac{i}{2} \partial^{\dot{\alpha}\alpha} A_\alpha,
\end{aligned} \tag{11}$$

where $\partial^{\dot{\alpha}\alpha} \equiv \bar{\sigma}^{\mu\dot{\alpha}\alpha} \partial / \partial x^\mu$. Using (6), we can write these superfields in terms of the world supercoordinates:

$$\begin{aligned}
W^\alpha(x, \theta, \bar{\theta}) &= -ie \int d\tau \left\{ \dot{\xi}^\alpha + i \dot{\xi}^\alpha \Delta \sigma^\mu \bar{\Delta} \partial_\mu + \frac{1}{4} \dot{\xi}^\alpha \Delta \Delta \bar{\Delta} \bar{\Delta} \square \right. \\
&\quad \left. + \omega_{\tau\mu} \left[2(\Delta \sigma^{\mu\nu})^\alpha \partial_\nu - \frac{i}{2} \Delta \Delta (\bar{\Delta} \bar{\sigma}_\nu)^\alpha (\partial^\mu \partial^\nu - \eta^{\mu\nu} \square) \right] \right. \\
&\quad \left. - i \Delta \Delta (\dot{\xi} \bar{\sigma}_\mu)^\alpha \partial^\mu \right\} \delta(s^2),
\end{aligned} \tag{12}$$

$$\bar{W}^{\dot{\alpha}}(x, \theta, \bar{\theta}) = (W^\alpha)^*.$$

It is convenient to use the superfields W and \bar{W} to introduce a superfield generalization of the Wheeler–Feynman electromagnetic current, by means of the equation

$$-4\pi \mathcal{J}(x, \theta, \bar{\theta}) = D^\alpha W_\alpha + \bar{D}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} = i \partial^{\dot{\alpha}\alpha} (\bar{D}_{\dot{\alpha}} A_\alpha - D_\alpha \bar{A}_{\dot{\alpha}}). \tag{13}$$

Under gauge transformations (9) we have $\delta \mathcal{J} = -i/16\pi [DD, \bar{D}\bar{D}] \Lambda = 0$, and the supercurrent \mathcal{J} is gauge-invariant. Using the explicit expressions for connections (6), we can rewrite the right side of (13) in the simple form $\square \Phi(x, \theta, \bar{\theta})$, where Φ is a scalar superfield of the type

$$\Phi(x, \theta, \bar{\theta}) = -4e \int d\tau [\omega_\tau^\mu (\Delta \sigma_\mu \bar{\Delta}) + i(\dot{\xi} \Delta) \bar{\Delta} \bar{\Delta} - i \Delta \Delta (\dot{\xi} \bar{\Delta})] \delta(s^2). \tag{14}$$

Using this observation and a fundamental relation for the interval in (3),

$$\square \delta(s^2) = -4\pi \delta^{(4)}(s^\mu), \tag{15}$$

which generalizes the Dirac identity⁶ to the supersymmetric case, we can write Eq. (13) as a superfield wave equation,

$$\square \Phi(x, \theta, \bar{\theta}) = -4\pi \mathcal{J}(x, \theta, \bar{\theta}), \tag{16}$$

with a supersymmetric electromagnetic current $\mathcal{J}(x, \theta, \bar{\theta})$,

$$\mathcal{J}(x, \theta, \bar{\theta}) = -4e \int d\tau [\omega_\tau^\mu (\Delta \sigma_\mu \bar{\Delta}) + i(\dot{\xi} \Delta) \bar{\Delta} \bar{\Delta} - i \Delta \Delta (\dot{\xi} \bar{\Delta})] \delta^{(4)}(s). \tag{17}$$

Equation (16) is a superfield generalization of Maxwell's equations in the Lorentz gauge. The physical meaning of the superfield $\Phi(x, \theta, \bar{\theta})$ is the prepotential⁴ $V(x, \theta, \bar{\theta})$ calculated in the gauge,

$$\{DD, \bar{D}\bar{D}\} V = 0 \Rightarrow V(x, \theta, \bar{\theta}) = \frac{1}{4} \Phi(x, \theta, \bar{\theta}), \tag{18}$$

which is a superfield generalization of the Lorentz gauge, which is fixed by the Wheeler–Feynman representation.¹ The justification for this treatment of the superfield Φ is that it is related to the superfields W_α and $\bar{W}_{\dot{\alpha}}$ in (11) by

$$W_\alpha = -\frac{1}{16} \bar{D} \bar{D} D_\alpha \Phi, \quad \bar{W}_{\dot{\alpha}} = -\frac{1}{16} D D \bar{D}_{\dot{\alpha}} \Phi, \quad (19)$$

which serve to define the prepotential.⁴ In the notation of Ref. 4, which we are using here, superfield condition (18) breaks up into the component gauge conditions

$$\begin{aligned} \square C(x) = -D(x), \quad \partial^{\dot{\alpha}\alpha} \chi_\alpha(x) = i \bar{\lambda}^{\dot{\alpha}}(x), \quad \partial^{\dot{\alpha}\alpha} \bar{\chi}_{\dot{\alpha}}(x) = -i \lambda^\alpha(x), \\ M(x) = N(x) = 0, \quad \partial_\mu v^\mu(x) = 0, \end{aligned} \quad (20)$$

which are imposed on the components C , χ , $\bar{\chi}$, M , N , and v of the gauge superfield $V(x, \theta, \bar{\theta})$. Conditions (20) make it possible to represent $V(x, \theta, \bar{\theta})$ by a component expansion

$$\begin{aligned} V(x, \theta, \bar{\theta}) = \frac{1}{4} \Phi(x, \theta, \bar{\theta}) = C + i \theta^\alpha \chi_\alpha - i \bar{\theta}_{\dot{\alpha}} \bar{\chi}^{\dot{\alpha}} - (\theta \sigma_\rho \bar{\theta}) v^\rho \\ + \frac{i}{2} \theta \bar{\theta} \bar{\theta}_{\dot{\alpha}} \bar{\lambda}^{\dot{\alpha}} - \frac{i}{2} \bar{\theta} \bar{\theta} \theta^\alpha \lambda_\alpha + \frac{1}{4} \theta \bar{\theta} \bar{\theta} D, \end{aligned} \quad (21)$$

where the vector field $v^\mu(x)$, the spinor fields $\lambda^\alpha(x)$ and $\bar{\lambda}^{\dot{\alpha}}(x)$, and the auxiliary field $D(x)$ form a Maxwell multiplet. The representations of these fields as integrals of the world supercoordinates are found through an expansion of prepotential Φ in (14) in powers of θ and $\bar{\theta}$. As a result, we find the following representation for the electromagnetic field $v^\mu(x)$, which generalizes Wheeler–Feynman field (1) and which is the zeroth term in the expansion of the vector connection $A_\mu(x, \theta, \bar{\theta})$ in (8):

$$\begin{aligned} v_\mu(x) \equiv i A_\mu(x, \theta=0, \bar{\theta}=0) = e \int d\tau \left\{ \dot{y}_\mu - \varepsilon_{\mu\nu\rho\lambda} \dot{y}^\nu (\xi \sigma^\rho \bar{\xi}) \partial^\lambda \right. \\ \left. + [\xi \bar{\xi} (\dot{\xi} \bar{\sigma}_{\mu\rho} \bar{\xi}) + (\dot{\xi} \sigma_{\mu\rho} \xi) \bar{\xi} \bar{\xi}] \partial^\rho + \frac{1}{4} \xi \bar{\xi} \bar{\xi} \dot{y}^\nu (\partial_\mu \partial_\nu - \eta_{\mu\nu} \square) \right\} \delta(s_0^2). \end{aligned} \quad (22)$$

It follows from this result in an obvious way that the Lorentz condition in (20), $\partial_\mu v^\mu = 0$, is satisfied. For the spinor field which is the superpartner of the field $v^\mu(x) = \partial^\mu v^\nu - \partial^\nu v^\mu$ we find

$$\begin{aligned} \lambda^\alpha(x) = e \int d\tau \left\{ \dot{\xi}^\alpha - i \dot{\xi} \bar{\xi} (\bar{\xi} \bar{\sigma}_\mu)^\alpha \partial^\mu + \frac{i}{2} \xi \bar{\xi} (\dot{\xi} \bar{\sigma}_\mu)^\alpha \partial^\mu - \frac{1}{2} \xi \bar{\xi} \xi \bar{\xi} \bar{\xi} \square \right. \\ \left. + \dot{y}_\mu \left[-2 (\xi \sigma^{\mu\nu})^\alpha \partial_\nu + \frac{i}{2} \xi \bar{\xi} (\bar{\xi} \bar{\sigma}_\nu)^\alpha (\partial^\mu \partial^\nu - \eta^{\mu\nu} \square) \right] \right\} \delta(s_0^2). \end{aligned} \quad (23)$$

The representation for $\bar{\lambda}^{\dot{\alpha}}$ is found through complex conjugation: $\bar{\lambda}^{\dot{\alpha}} = (\lambda^\alpha)^*$. For the auxiliary field $D(x)$ we find

$$D(x) = e \int d\tau \left\{ \dot{y}_\mu (\xi \sigma^\mu \bar{\xi}) - i [\xi^2 (\bar{\xi} \bar{\xi}) - (\dot{\xi} \bar{\xi}) \bar{\xi}^2] \square \right\} \delta(s_0^2). \quad (24)$$

The fields $\lambda^\alpha(x)$ and $\bar{\lambda}_{\dot{\alpha}}(x)$ in (23) and v^μ in (22), which are constructed from the world supercoordinates, satisfy wave equations for Maxwell and Weyl fields with currents on their right sides:

$$\square v^\mu(x) = -4\pi j^{(2)\mu}(x), \quad \partial_{\alpha\dot{\alpha}}\bar{\lambda}^{\dot{\alpha}}(x) = -4\pi j_\alpha^{(1)}(x), \quad \partial^{\dot{\alpha}\alpha}\lambda_\alpha(x) = -4\pi \bar{j}^{(1)\dot{\alpha}}(x), \quad (25)$$

and $D(x) = -4\pi j^{(0)}$. Superfield equation (16) decomposes into these equations when we substitute in representation (14) and a component decomposition of the supercurrent of the form

$$\begin{aligned} \mathcal{J} = & -4j^{(0)} + 4\theta^\alpha j_\alpha^{(1)} - 4\bar{\theta}_{\dot{\alpha}} \bar{j}^{\dot{\alpha}(1)} - 4(\theta\sigma_\rho\bar{\theta})j^{(2)\rho} \\ & - 2i\theta\bar{\theta}\bar{\theta}_{\dot{\alpha}}\partial^{\dot{\alpha}\alpha}j_\alpha^{(1)} + 2i\bar{\theta}\bar{\theta}\theta^\alpha\partial_{\alpha\dot{\alpha}}\bar{j}^{(1)\dot{\alpha}} + \theta\bar{\theta}\bar{\theta}\square j_0. \end{aligned} \quad (26)$$

In turn, the explicit expression for the components (26) of the current multiplet $\mathcal{J}(x, \theta, \bar{\theta})$ is found from integral representations (22) and (23) after we apply differential d'Alembertian and Dirac operators to them. As a result, we find the following representation for the electromagnetic current:

$$\begin{aligned} j_\mu^{(2)} = & e \int d\tau \left\{ \dot{y}_\mu - \varepsilon_{\mu\nu\rho\lambda} \dot{y}^\nu (\xi\sigma^\rho\bar{\xi}) \partial^\lambda + \frac{1}{4} \xi\bar{\xi}\bar{\xi}\dot{y}^\nu (\partial_\mu\partial_\nu - \eta_{\mu\nu}\square) \right. \\ & \left. + [\xi\bar{\xi}(\bar{\xi}\bar{\sigma}_{\mu\rho}\bar{\xi}) + (\xi\sigma_{\mu\rho}\xi)\bar{\xi}\bar{\xi}] \partial^\rho \right\} \delta^{(4)}(s_0). \end{aligned} \quad (27)$$

This representation is a supersymmetric generalization of the electromagnetic current in the Wheeler–Feynman approach. Conservation of electromagnetic current ($\partial^\mu j_\mu^{(2)} = 0$) follows from explicit expression (27). For the spinor component of the supercurrent \mathcal{J} in (26) we find

$$\begin{aligned} j_\alpha^{(1)} = & e \int d\tau \left\{ \dot{y}_\mu \left[(\sigma^\mu\bar{\xi})_\alpha - \frac{i}{2} \xi_\alpha\bar{\xi}\bar{\xi}\partial^\mu - i(\sigma^{\mu\rho}\xi)_\alpha\bar{\xi}\bar{\xi}\partial_\rho \right] \right. \\ & \left. + i\dot{\xi}_{-\alpha}\bar{\xi}\bar{\xi} - \frac{1}{2}(\sigma^\rho\bar{\xi})_\alpha\xi\bar{\xi}\bar{\xi}\partial_\rho \right\} \delta^{(4)}(s_0). \end{aligned} \quad (28)$$

We find the corresponding expression for $\bar{j}_{\dot{\alpha}}^{(1)}$ through complex conjugation: $\bar{j}_{\dot{\alpha}}^{(1)} = (j_\alpha^{(1)})^*$.

Expressions (22)–(28) for the fields and currents can be simplified by using the Dirac identity $\square\delta(s_0^2) = -4\pi\delta^{(4)}(s_0)$.

The introduction of Grassmann variables in the theory of action at a distance makes it possible to incorporate the contribution of spin degrees of freedom of the charged particles in the classical limit, $\hbar \rightarrow 0$. This effect arises even in the static approximation, with $\dot{y} = \dot{\xi} = \dot{\bar{\xi}} = 0$. In this case, the components of the electromagnetic 4-potential $v^\mu = (v_0, \mathbf{v})$ can be written in the following form after a direct evaluation of the integrals in (22) in the gauge $\tau = y_0$:

$$v_0 = \frac{e}{r} + e\pi\xi_0^2\bar{\xi}_0^2\delta^{(3)}(\mathbf{r}), \quad \mathbf{r}=\mathbf{x}-\mathbf{y},$$

$$\mathbf{v} = e \frac{[\boldsymbol{\Sigma} \times \mathbf{r}]}{r^3}, \quad \boldsymbol{\Sigma} = (\xi_0 \boldsymbol{\sigma} \bar{\xi}_0), \quad \xi_0^\alpha = \xi_0^\alpha \Big|_{r=x_0}. \quad (29)$$

It can be seen from (22) that we obtain a nonvanishing 3-vector potential \mathbf{v} , in contrast with the Wheeler–Feynman results. This vector potential is generated by the magnetic moment $\boldsymbol{\Sigma}$ of the particle, which is proportional to the expectation value of the operator representing the spin of the source particle, $\boldsymbol{\sigma}$.

A second physical effect which stems from the introduction of Grassmann variables is the addition of a term proportional to $\delta^{(3)}(\mathbf{r})$ in the scalar potential v_0 . To clarify the physical meaning of this term, we apply a Laplacian to the equation for v_0 in (29):

$$\Delta v_0 = -4\pi \left[\frac{e}{2} \delta^{(3)}(\mathbf{r} + \boldsymbol{\Sigma}) + \frac{e}{2} \delta^{(3)}(\mathbf{r} - \boldsymbol{\Sigma}) \right]. \quad (30)$$

It can be seen from the expression for the charge density on the right side of (30) that incorporating the spin of the particle leads to a “smearing” of the point charge e of the particle over a spatial region with the scale of the Compton wavelength. This effect of the nonzero size of charged superparticles stems from the creation of electron–positron pairs (Zitterbewegung), which does not disappear in the limit under consideration here, $\hbar \rightarrow 0$, since the contribution of the spin degrees of freedom of the particle does not vanish in this limit.

As can be seen from representation (22), going beyond the scope of the static approximation makes it possible to incorporate the contribution of the spin–orbit interaction and subsequent terms in the expression for the Hamiltonian of the interaction of the charged fermions,⁹ written as a power series in v/c .

We wish to thank A. I. Akhiezer, D. V. Volkov, S. V. Peletminskiĭ, I. A. Bandos, A. P. Rekalov, and Yu. P. Stepanovskii for a valuable discussion and for critical comments. This study was supported in part by grant RY9000 of the Soros International Science Foundation and also by the Foundation of the Ukrainian State Committee on Science and Technologies, in the Fundamental Research Program.

¹⁾All the differential operators ∂ , D , and \square used in this paper act on the coordinates $(x, \theta, \bar{\theta})$ of the “observation point.”

¹J. A. Wheeler and R. P. Feynman, *Rev. Mod. Phys.* **21**, 425 (1949).

²A. A. Zheltukhin and V. V. Tugaĭ, *JETP Lett.* **58**, 252 (1993).

³D. V. Volkov and V. P. Akulov, *JETP Lett.* **16**, 438 (1972).

⁴J. Wess and J. Bagger, *Supersymmetry and Supergravity* (Princeton U. Press, Princeton, N.J., 1983).

⁵V. I. Ogievetskiĭ and L. Mezincesku, *Usp. Fiz. Nauk* **117**, 637 (1975) [*Sov. Phys. Usp.* **18**, 960 (1975)].

⁶P. A. M. Dirac, *Proc. R. Soc. A* **167**, 148 (1938).

⁷M. Kalb and P. Ramond, *Phys. Rev. D* **9**, 2273 (1974).

⁸M. B. Green *et al.*, *Superstring Theory* (Cambridge U. Press, Cambridge, 1987), Vol. 1 and 2.

⁹A. I. Akhiezer and V. B. Berestetskiĭ, *Quantum Electrodynamics* [in Russian] (Nauka, Moscow, 1969).

Translated by D. Parsons