Theory of a diffuse domain wall

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A theory of a diffuse domain wall is proposed. This theory agrees satisfactorily with experimental results. © *American Institute of Physics.*

I. INTRODUCTION

In the course of magnetooptic observations of the motion of the wall of a magnetic bubble (or cylindrical magnetic domain) being expanded by a pulse of a bias field, Zimmer *et al.*[1] observed a smearing of the wall. The effect has come to be known as a "diffuse domain wall." This phenomenon has since then been studied by many investigators, in various scientific laboratories; it has been observed in magnetic films with various properties.[2–6] Despite the fairly extensive experimental results which have been found, this phenomenon has yet to be explained.

Let us review some characteristic experimental features of diffuse domain walls. In most cases, the wall arises when there is a magnetic field in the plane of the film or when there is a planar anisotropy. A broadening of the domain wall occurs only in regions oriented perpendicular to this field. The broadening occurs up to a certain maximum value; beyond this point, the wall moves with a constant effective diffuse width, in the manner of a normal domain wall, until the end of the pulses of the bias field H_0. The value of H_0 at which the diffuse wall is observed is $4\pi M_s$ in order of magnitude. The time scale of the formation of a diffuse wall is several tenths of a microsecond. An in-plane field promotes the onset of a diffuse wall, but it is apparently not a necessary condition for the onset of a wall, since a wall was observed in Ref. 5 even in the absence of an in-plane field or a corresponding anisotropy. In this letter we propose a theory for diffuse domain walls which explains these experimental features and which predicts satisfactory quantitative results.

II. THEORY

For magnetic-film materials with a large quality factor, under the condition $K \gg 2\pi M^2$, where K is the uniaxial-anisotropy constant, and M the magnetization vector, we can use the system of Slonczewski equations, which follow from the Landau-Lifshitz micromagnetic equations:[6,8]

$$q_t = \gamma\Delta[f(\varphi) - (2A/M)\varphi_{zz}] + \alpha\Delta\varphi_t, \tag{1}$$

$$\varphi_t = \gamma[H_0 + (2A/M)q_{zz}/\Delta] - \alpha q_t/\Delta, \tag{2}$$

$$f(\varphi) = \pi[H_x \sin\varphi - (H_y + H_s(z)] \cos\varphi) + 2\pi M \sin 2\varphi. \tag{3}$$

Here $q(t,z)$ is the coordinate at the middle of the wall, at a height z at a time t; the Z coordinate axis is perpendicular the plane of the film; $\varphi(t,z)$ is the azimuthal angle of the magnetization vector M; $\Delta=(A/K)^{1/2}$ is the width of the wall; $H_s(z)$ is the twisting field; and H_x and H_y are components of the in-plane field.

A diffuse wall is observed at bias fields H_0 which are much stronger than the Walker critical field $H_w=\alpha 2\pi M$. In this case, the spins in the wall precess at an angular velocity on the order of γH_0 (Ref. 8). Taking an average of Eqs. (1)–(3) over the period of these oscillations, we find, in the corresponding approximation,

$$q_t=\alpha\Delta\varphi_t,\quad (1+\alpha^2)\varphi_t=\omega_H-(a^2+b^2)/2\omega_H+(2\gamma A/M)q_{zz}/\Delta, \tag{4}$$

where $\omega_H=\gamma H$,

$$a=\pi\alpha\gamma[H_y+H_s(z)],\quad b^2=(\alpha\gamma 2\pi M)^2+(\alpha\gamma\pi H_x/2)^2. \tag{5}$$

System of equations (4) has a steady-state solution in the case $\varphi_t=\omega=\text{const}$, which corresponds to uniform motion of the average domain wall. In this case we find the following expression from (4):

$$q_{zz}=a_1+a_2H_s(z)+a_3H_s(z)^2, \tag{6}$$

where

$$a_1=\Delta\frac{M}{2\gamma A}\left[(1+\alpha^2)\omega-\omega_H+\frac{\alpha^2\gamma^2\pi^2}{2}\frac{H_y^2}{2\omega_H}+\frac{b^2}{2\omega_H}\right],$$
$$a_2=\Delta\frac{M}{2\gamma A}\frac{\alpha^2\gamma^2\pi^2}{4}\frac{H_y}{2\omega_H},\quad a_3=\Delta\frac{M}{2\gamma A}\frac{\alpha^2\gamma^2\pi^2}{8\omega_H}. \tag{7}$$

Under the free boundary conditions

$$q_z(0)=q_z(h)=0, \tag{8}$$

a solution of Eq. (6) is

$$q(z)=a_2\int_0^z dz_1\int_0^{z_1}H_s(z_2)dz_2+a_3\int_0^z dz_1\int_0^{z_1}H_s(z_2)^2dz_2$$
$$-\frac{a_3z^2}{2h}\int_0^h H_s(z)^2dz+q(0). \tag{9}$$

In deriving (9) we used the equality

$$a_1=-a_3\frac{1}{h}\int_0^h H_s(z)^2dz\equiv -a_3\langle H_s(z)^2\rangle, \tag{10}$$

which follows from conditions (8).

It is a simple matter to work from (10) to derive an expression for the angular precession velocity of the spins, ω, and for the displacement velocity of the domain wall, q_t, in the course of an average steady-state motion of the wall:

$$\omega=\left[1+\frac{\alpha^2\gamma^2\pi^2}{4\omega_H^2}\left(H_y^2+\frac{\langle H_s(z)^2\rangle}{2}\right)-\frac{b^2}{2\omega_H^2}\right]\frac{\omega_H}{1+\alpha^2}, \tag{11}$$

$$q_t=\alpha\Delta\omega. \tag{12}$$

Approximating the twisting field by a linear function

$$H_s(z)=4\pi M(2z/h-1), \tag{13}$$

we find the following expression for the bending of the wall, (9):

$$q(z)=W_0\left\{2x^3-3x^2+\frac{1}{16}\frac{4\pi M}{H_y}[(2x-1)^2-1]^2\right\}+q(0), \tag{14}$$

where $x=z/h$, $\Lambda_0^2=A/2\pi M^2$, and

$$W_0=\Delta(\pi\alpha h/\Lambda_0)^2(H_y/H_0)/24. \tag{15}$$

In the course of steady-state motion, the wall is thus in a bent state; the shape of the bend is determined by (9) or (14). At $H_y \sim 4\pi M$ the second term in (14) is an order of magnitude weaker than the first, so the bending of the wall is determined by the magnetic field H_y, which is directed perpendicular to the plane of the wall. In this approximation, the field H_x does not affect the bending of the wall. The maximum bending of the wall, determined by the field H_y, is

$$W=q(h)-q(0)=W_0. \tag{16}$$

In the absence of an in-plane field we find the following expression for the maximum bend of a moving wall, working from (14):

$$W=q(h/2)-q(0)=\Delta(\pi\alpha h/\Lambda_0)^2(4\pi M/H_0)/384. \tag{17}$$

To find the time scale over which the wall acquires its steady-state shape, we need to solve time-dependent system of equations (4). For this purpose we put Eqs. (4) in the form

$$q_t=\kappa^2 q_{zz}+f(z), \tag{18}$$

where

$$\kappa^2=\frac{\alpha}{1+\alpha^2}\frac{2\gamma A}{M}, \tag{19}$$

$$f(z)=\frac{\alpha\Delta\omega_H}{1+\alpha^2}\left(1-\frac{a^2+b^2}{2\omega_H^2}\right). \tag{20}$$

Solving Eq. (18) by separation of variables under the initial condition $q(0,z)=0$ and under boundary conditions (8), we find

$$q(t,z)=f_0 t+\sum_{n=1}^{\infty}[1-\exp(-\kappa^2k_n^2t)]\frac{2f_n}{h\kappa^2k_n^2}\cos(k_n z), \tag{21}$$

where

$$k_n = \pi n/h, \quad n = 1,2,3,..., \tag{22}$$

$$f_n = \frac{2}{h}\int_0^h f(z)\cos(k_n z)dz, \quad f_0 = \frac{1}{h}\int_0^h \dot{f}(z)dz. \tag{23}$$

It is not difficult to see that the quantity f_0 is the same as the velocity of the steady-state average motion of the wall, (12), and that the series in (21) becomes a Fourier expansion of a steady-state bend, (8) or (14), in the limit $t \to \infty$.

It follows from (21) that the time scale (τ) over which the wall reaches a steady-state bend as in (8) is, in order of magnitude,

$$\tau = \frac{1}{\kappa^2 k_1^2} = \frac{1+\alpha^2}{\alpha}\frac{M}{2\gamma A}\frac{h^2}{\pi^2} = \left(\frac{h}{\pi\Lambda_0}\right)^2 \frac{1}{\alpha\omega_M}, \tag{24}$$

where $\omega_M = \gamma 4\pi M$.

III. COMPARISON WITH EXPERIMENTAL RESULTS

Under the assumption that a diffuse domain wall arises in the course of the motion of a domain wall because of the bending of the wall discussed in the preceding section of this letter, let us compare (on the one hand) the maximum wall broadening in (14) and the time taken to establish this broadening, (24), with (on the other) the experimental results of Refs. 1–5. First, however, we note that the damping parameter αgiven in Refs. 1, 3, and 4 was based on ferromagnetic-resonance data. For rare-earth iron garnets, the dynamic damping parameter is[6] usually an order of magnitude larger than the value found for α on the basis of ferromagnetic-resonance data. We will accordingly estimate αon the basis of data on the mobility of the wall given in those papers.

As a result, for the wall broadening W and the time τ we find the following results from Eqs. (16) and (24), using the parameters of the sample from Ref. 2: $W = 19$ μm and $\tau = 1.9$ μs (the experimental results are $W = 12$ μm and $\tau = 0.5$ μs). For the parameters of the sample from Ref. 3, with $\alpha = 0.3$, we find $W = 5.4$ μm and $\tau = 0.6$ μs (the experimental results are 4 μm and 0.4 μs). For the parameters of the sample from Ref. 1, with $\alpha = 0.35$, we find $W = 9.8$ μm and $\tau = 2.1$ μs (the experimental results are 25 μm and 0.4 μs). For the parameters of the sample from Ref. 4, with $\alpha = 0.5$, we find $W = 3.5$ μm and $\tau = 0.17$ μs (the experimental results are 4 μm and 0.15 μs).

Kleparski *et al.*[5] observed a diffuse domain wall along the entire perimeter of an expanding magnetic bubble in the case $H_y = 0$. Using the wall mobility curve given in Ref. 5, we find an estimate $\alpha \sim 0.5$ of the damping parameter. From Eqs. (16) and (24) we then find $W = 53$ μm and $\tau = 1$ μs for the sample of Ref. 5 (the experimental results are $W = 20$ μm and $\tau = 2$ μs).

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