Distinctive properties of 1D normal-metal–superconductor contacts for superconductors with a broken particle–hole symmetry

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The time-dependent Ginzburg–Landau equation with a complex relaxation constant $\gamma=\gamma_0(1-i\epsilon)$ is used to analyze distinctive features in the properties of 1D normal-metal–superconductor contacts near T_c. The current–voltage characteristics of the contact are asymmetric in this model. The asymmetry depends strongly on $T-T_c$ and also the sign of ϵ. A value $\epsilon\neq 0$ has been invoked previously to explain a change in the sign of the Hall conductivity of a high T_c superconductor in the mixed state. The results found in the present study indicate that it may be possible to determine the absolute value and the sign of ϵ from the temperature dependence of the asymmetry of the current–voltage characteristics of normal-metal–high-T_c-superconductor contacts near T_c. This capability would be of importance for, in particular, interpreting Hall-effect experiments. © *1994 American Institute of Physics.*

There has recently been an extensive discussion in the literature of whether certain distinctive features of the dynamic properties of high-T_c superconductors can be described by a time-dependent Ginzburg–Landau equation with a complex relaxation constant[1-3] $\gamma=\gamma_0(1-i\epsilon)$:

$$-\gamma\left(\hbar\frac{\partial}{\partial t}+2ie\varphi\right)\Psi=\frac{\delta F_s}{\delta\Psi^*}, \qquad (1)$$

where F_s is the Ginzburg–Landau free energy. The nonzero imaginary part of γ violates the symmetry of Eq. (1) under the transformation $\Psi\rightarrow\Psi^*$, $\varphi\rightarrow-\varphi$, $\mathbf{A}\rightarrow-\mathbf{A}$. This circumstance corresponds to a breaking of particle–hole symmetry.[2] The derivation of a time-dependent equation like (1) from a microscopic theory had been taken up previously by Ebisawa and Fukuyama.[4] They found $\epsilon\sim T_c/E_F$, so this quantity is quite small (although this ratio is considerably larger for high-T_c superconductors than for most low-temperature superconductors). Nevertheless, the incorporation of ϵ leads to qualitative changes in both the dynamics of the order parameter and the distribution of the potential φ. It was shown in Refs. 1–3 that this approach can explain experimental data which indicate a change in the sign of the Hall conductivity in high-T_c superconductors (see, for example, Refs. 5–10). The sign of the quantity ϵe is a very important question, since a change in the sign of the Hall conductivity becomes possible under the condition sign $(\epsilon e)=-\mathrm{sign}\sigma_n^H$ (σ_n^H is the Hall conductivity in the normal state). As was mentioned in Refs. 2 and 3, this condition can be realized for superconductors with a complex Fermi surface [for a simple, isotropic Fermi surface we would have $\mathrm{sign}(\epsilon e)=\mathrm{sign}\sigma_n^H$]. It would clearly be of interest to determine other possible consequences of values $\epsilon\neq 0$. As one of these consequences, we discuss in this letter the effect of a breaking of particle–hole symmetry on the properties of 1D normal-metal–superconductor (N–S) contacts. The current–voltage characteristics of N–S contacts were analyzed in the ordinary time-dependent Ginzburg–Landau theory ($\epsilon=0$) in Refs. 11 and 12. Here we use a system of time-dependent Ginzburg–Landau equations for superconductors with a finite gap.[13–15] In this system we incorporate an imaginary increment in the relaxation constant:

$$\frac{\pi\hbar}{8(T_c-T)}\left(1+\frac{|\Psi|^2}{\Gamma^2}\right)^{-1/2}\left(\frac{\partial}{\partial t}+\frac{2ie\varphi}{\hbar}+\frac{1}{2\Gamma^2}\frac{\partial|\Psi|^2}{\partial t}\Psi-i\epsilon\frac{\pi\hbar}{8(T_c-T)}\right.$$
$$\left.\times\left(\frac{\partial}{\partial t}+\frac{2ie\varphi}{\hbar}\right)\Psi=\xi^2\left(\nabla-\frac{2ie}{\hbar c}\mathbf{A}\right)^2\Psi+\Psi-\left|\Psi\right|^2\Psi, \tag{2}$$

$$\mathbf{j}=\sigma\mathbf{E}+\mathbf{j}_s, \tag{3}$$

where $\xi(\mathrm{T})$ is the coherence length, $\mathbf{j}_s$ is the superconducting current density, we have

$$\Gamma^{-1}=(4\tau_{\mathrm{ph}}\pi/\hbar)\sqrt{2T_c(T_c-T)/[7\zeta(3)]},$$

and τ_{ph} is the inelastic electron–phonon relaxation time. These equations can be derived from a microscopic theory under the condition $T_c-T\ll\hbar/\tau_{\mathrm{ph}}$ (for the case $\epsilon=0$ see the derivation in Refs. 13–15; for the case $\epsilon\neq 0$, this question will be taken up in a separate paper). In the limit $\Gamma\to\infty$, Eq. (2) yields Eq. (1), which corresponds to the gapless case. Let us consider a 1D system consisting of a superconductor ($x>0$) and a normal metal ($x<0$). We seek steady-state solutions for Ψ as a small direct current flows across the boundary ($j\ll j_c$, where j_c is the density of the critical depairing current). System (2), (3) can then be written

$$\xi^2\frac{\partial^2\rho}{\partial x^2}+\left(1-\frac{\pi\epsilon e}{4(T_c-T)}\varphi\right)\rho-\rho^3-\xi^2\rho\left(\frac{\partial\theta}{\partial x}\right)^2=0, \tag{4}$$

$$\xi^2\frac{\partial^2\varphi}{\partial x^2}=u\rho^2\varphi\left(1+\frac{\rho^2}{\Gamma^2}\right)^{-1/2}, \tag{5}$$

$$j=-\sigma\frac{\partial\varphi}{\partial x}+j_s, \tag{6}$$

where ρ and θ are the amplitude and phase of the order parameter. The boundary conditions on this system of equations are

$$\rho(0)=0,\quad \rho(+\infty)=1,\quad \frac{\partial\varphi}{\partial x}(0)=-j/\sigma,\quad \varphi(+\infty)=0.$$

The parameter u determines the ratio of the coherence length to the penetration depth for the electric field. Its value can be found from the microscopic theory:[13–15] $u=5.79$. For a superconductor with a high magnetic-impurity concentration, we need to take the limit $\Gamma\to\infty$ and set $u=12$ in Eqs. (2) and (5). It follows from Eq. (4) that the potential φ either suppresses or increases the modulus of the order parameter near the N–S boundary, depending on the sign of the product $\epsilon e\varphi$. As a result, the current–

voltage characteristic is asymmetric. We seek a solution of Eqs. (4) and (5) as a power series in the small quantity j/j_c. Retaining only the terms linear and quadratic in j, we find the following expression for the current–voltage characteristic:

$$V \simeq \frac{\xi\alpha_1}{\sigma} j \left[1+\epsilon \operatorname{sign}(e)\alpha_2 \frac{j}{j_c} \right], \tag{7}$$

where V is the potential difference between a point at the N–S interface and a point in the interior of the superconductor. Here we will discuss the two limiting cases (a) $\Gamma \gg 1$ and (b) $\Gamma \ll 1$, $u\Gamma \ll 1$. If $\Gamma \gg 1$, and Eq. (2) becomes (1), the functions α_1 and α_2 can be found from the expressions

$$\alpha_1 = f(0,u), \tag{8}$$

$$\alpha_2 = -\frac{4u^2}{3\sqrt{3}f(0,u)} \int_0^\infty f^2(z,u) R(z) \tanh \frac{z}{\sqrt{2}}\, dz, \tag{9}$$

$$R(z) = y_2(z) \int_z^\infty y_1(z') f(z',u) \tanh \frac{z'}{\sqrt{2}}\, dz' + y_1(z) \int_0^z y_2(z') f(z',u) \tanh \frac{z'}{\sqrt{2}}\, dz', \tag{10}$$

$$y_1(z) = \cosh^{-2} \frac{2}{\sqrt{2}}, \quad y_2(z) = y_1(z) \int_0^z \frac{dz'}{y_1^2(z')}. \tag{11}$$

The function $f(z,u)$ is a solution of the equation

$$\frac{\partial^2 f}{\partial z^2} = u \tanh^2 \frac{z}{\sqrt{2}} f, \tag{12}$$

$$\frac{\partial f}{\partial z}(0,u) = -1, \quad f(\infty,u) = 0.$$

It is of the form $f(z,u) = -\eta(z)/n_z'(0)$, where

$$\eta(z) = \left(\cosh \frac{z}{\sqrt{2}} \right)^{-\sqrt{2u}} F\left[\sqrt{2u}-s,\ \sqrt{2u}+s+1,\ \sqrt{2u}+1,\ \left(1-\tanh \frac{z}{\sqrt{2}} \right)/2 \right], \tag{13}$$

$$s = 0.5(-1+\sqrt{1+8u}).$$

Here $F(\alpha,\beta,\gamma,x)$ is the hypergeometric function. The quantities α_1 and α_2 have been found numerically for various values of the parameter u. For $u=5.79$ we find $\alpha_1 \simeq 1.2$, $\alpha_2 \simeq 0.27$; for $u=12$ we find $\alpha_1 \simeq 1$, $\alpha_2 \simeq 0.33$.

We now consider the case $\Gamma \ll 1$, $u\Gamma \ll 1$. In this case, the depth to which an electric field penetrates into the superconductor is large in comparison with ξ. Under the condition $x \gg \xi$ we can ignore the term $\xi^2 \rho_{xx}''$ in Eq. (4). Considering only the component which is linear in j, we find the following expression for the modulus of the order parameter in the region $x \gg \xi$:

$$\rho \simeq 1 - \frac{j\xi}{\sigma} \frac{\pi \epsilon e}{8\sqrt{u\Gamma}(T_c - T)} \exp\left(-\frac{x\sqrt{u\Gamma}}{\xi} \right). \tag{14}$$

Using (5) and (14), we find the correction to the potential $\varphi(x)$ which is quadratic in j, and for $V(j)$ we find an expression of the type in (7), where

$$\alpha_1 = \frac{1}{\sqrt{u\Gamma}}, \quad \alpha_2 = \frac{1}{9}\sqrt{\frac{u}{3\Gamma}}. \tag{15}$$

A term quadratic in the current in the expression for $V(j)$ is characteristic of specifically a superconductor with a broken particle–hole symmetry. In the case $\epsilon=0$, there is a deviation from a linear $V(j)$ law because of terms on the order of j^3/j_c^2 (Ref. 12). The current–voltage characteristic in (7) is asymmetric. The magnitude of the asymmetry depends strongly on $T-T_c$, and it increases with increasing value of the ratio $\epsilon/\sqrt{\Gamma}$. The asymmetry also depends on the sign of the product ϵe. Experimental observation of this asymmetry would make it possible to determine the sign of ϵ. This information would be important, in particular, for interpreting experiments on the Hall effect in high-T_c superconductors[5–10] [the sign of the correction to the Hall conductivity at $H<H_{c2}$ is also determined by the sign of the quantity $\epsilon \text{sign}(e)$; Ref. 3].

We note in conclusion that the question of the flow of a direct current across an N–S interface and the question of the nonlinearity of the current–voltage characteristic have also been analyzed in the microscopic theory in a temperature interval near T_c, in which the condition for the applicability of the equations used above, $T_c - T \ll \hbar/\tau_{\text{ph}}$, is violated in superconductors with a finite gap (see Refs. 16 and 17 and the papers cited there). In particular, a mechanism for a nonlinearity of the current–voltage characteristic involving the effect of the current on the relaxation rate of the electron–hole imbalance was studied in Ref. 17. That effect, however, turns out to be small in the temperature interval discussed above,[17] $T_c - T \ll \hbar/\tau_{\text{ph}}$. Moreover, that effect does not lead to an asymmetry of the current–voltage characteristic, in contrast with the nonlinearity mechanism proposed in the present letter.

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