Nature of the anomalous thermal conductivity of high-T_c superconductors

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The anomalous increase in the thermal conductivity below the superconducting transition temperature T_c of high-T_c superconductors may be due to the existence of weakly damped collective electron excitations of a Bose type with an acoustic dispersion relation (acoustic plasmons) inside the superconducting gap $2\Delta(T)$. © *1994 American Institute of Physics.*

1. In ordinary, low-temperature superconductors, the electron component of the thermal conductivity falls off rapidly with decreasing temperature below T_c because normal Fermi excitations become "frozen."[1,2] The phonon component of the thermal conductivity increases because the electron mechanism for the absorption of phonons inside the superconducting gap turns off. The resultant thermal conductivity of the superconductor may either decrease or increase at $T<T_c$, depending on the purity of the sample, the structure of its electron and phonon spectra, etc.

The high-temperature superconductors (high-T_c materials) based on layered cuprate metal oxides exhibit a sharp increase in thermal conductivity below T_c in the plane of the CuO_2 layers.[3–7] This conductivity reaches a maximum at $T\approx T_c/2$. At its maximum, the thermal conductivity may exceed that in the normal state, at $T\geqslant T_c$, by a factor of 1.5 or 2. This increase in the thermal conductivity is difficult to explain on the basis of an increase in the lattice component alone, as was suggested in Refs. 8 and 9, for example. The difficulty is that, despite the quasi-2D nature of the electron spectrum and the pronounced anisotropy of the electron–phonon relaxation, this mechanism leads to an increase in the thermal conductivity at $T<T_c$ not only in the a–b plane but also along the **c** axis.[8] This behavior contradicts the experiments of Refs. 3 and 4, according to which there is no peak in the thermal conductivity along the **c** direction, and the lattice component of the thermal conductivity in the a–b plane does not exceed 30% of the maximum thermal conductivity.[6,10] Furthermore, the thermal conductivity along the **c** axis is smaller than that in the a–b plane by at least an order of magnitude.[3] It is thus natural to link the anomalous increase in the thermal conductivity along the conducting CuO_2 layers in layered cuprates below T_c with a nonphonon component of the thermal conductivity.[7,8]

The maximum in the thermal conductivity was thus explained in Ref. 11 on the basis of a Bose condensation of a 2D Bose gas of bipolarons at $T<T_c$ with a bipolaron mechanism for the high-T_c superconductivity.[12,13] It was suggested in Ref. 11 that the small-radius bipolarons in the cuprate layers have a square-root spectrum, $\omega_k\sim\sqrt{k}$, in the limit $k\to 0$, as in 2D systems. This behavior would lead to a $k^{-3/2}$ singularity as $k\to 0$ in the momentum dependence of the transport scattering times. As a result, the increase in thermal conductivity with decreasing T has a $(T_c-T)^{3/2}$ behavior near T_c in the bipolaron

model, while the decrease in the thermal conductivity at low T is cubic ($\sim T^3$), as in the case of the phonon component of the thermal conductivity.

In this letter we show that the sharp increase in the thermal conductivity below T_c may be due to (on the one hand) the existence of a branch of low-frequency collective excitations of the electron density of the Bose type, with an acoustic dispersion relation (acoustic plasmons),[14] and (on the other) a suppression of quantum Landau damping for such excitations inside the gap $2\Delta(T)$ in the spectrum of quasiparticles at $T<T_c$. This mechanism for the thermal conductivity leads to a decrease in the thermal conductivity which is quadratic in T as $T\rightarrow 0$ (because of the quasi-2D nature of this spectrum of acoustic plasmons), while it leads to a linear increase, $\sim(T_c-T)$, in the thermal conductivity below T_c, in good agreement with experiment.[3–6]

2. It has been suggested that a branch of acoustic plasmons exists in the layered cuprate compounds because there are overlapping wide and narrow 2D bands in their electron spectra near the Fermi level. This suggestion was first made in Refs. 12 and 15. These bands would be partially filled with light (l) and heavy (h) carriers (electron and holes). This suggestion is supported by the weak dependence of the position of the Fermi level and of the optical plasma frequency $\Omega_{\rm pl}$ in $La_{2-x}(Ba, Sr)_xCuO_4$ on the degree of doping.[16,17] This circumstance can be explained on the basis of a predominant filling of an anomalously narrow band with a high density of states and a large effective mass of the h carriers. The density of the l carriers, N_l, in the wide 2D band, with a small density of states, is essentially unaffected by the doping. Consequently, their Fermi energy $E_{Fl}\sim N_l$ and the plasma frequency $\Omega_l\sim\sqrt{N_l}$ remain essentially constant, while the variable plasma frequency of the h carriers satisfies $\Omega_h\ll\Omega_l$ in the entire variation range of x; we thus have $\Omega_{\rm pl}=(\Omega_l^2+\Omega_h^2)^{1/2}\approx$const.

Direct observation of flat (dispersion-free) bands near symmetry points of the Brillouin zone in p-type cuprates has recently been reported.[18] According to the experimental data of Ref. 18 and also according to numerical calculations of the band spectrum (Refs. 19–21, for example), the cylindrical Fermi surface of the layered cuprates is multiply connected (is of a multivalley nature) within the first Brillouin zone, and the electron states of different bands (valleys) are separated in momentum space because of the low density of doped l and h carriers ($N_l\approx N_h\lesssim 10^{14}$ cm^{-2} in each 2D CuO_2 layer). In this case, a model of a multicomponent charged Fermi liquid with l and h carriers is a good approximation for describing collective electron excitations. It follows immediately from this model that acoustic plasmons[14] exist as "Goldstone excitations" of a sort, associated with a "spontaneous" violation of the principle of the indistinguishability of particles for electrons (holes) in different bands (valleys). In addition, at sufficiently high temperatures, $T>W_h/4$, where W_h is the width of the narrow 2D band ($W_h<45$ MeV, according to Ref. 18), h carriers which are nearly localized at lattice sites become nondegenerate, so the relationship between the spin and the statistics is disrupted, and the filling of the narrow band can be described approximately by a Maxwell–Boltzmann distribution function. As was shown in Refs. 22 and 23, a multicomponent Fermi liquid has marginal properties.[24]

In the normal metallic state, the branch of acoustic plasmons with a frequency $\omega_q\approx qu$ lies in a region of quantum Landau damping by l carriers with a damping rate

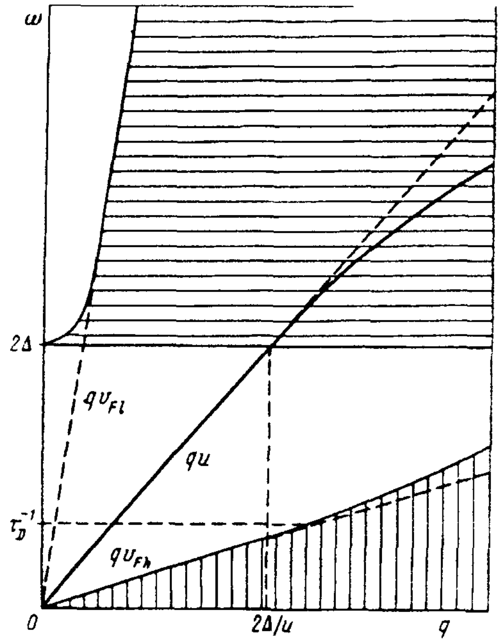

FIG. 1. Spectrum of acoustic plasmons (the heavy solid curve) and regions of quantum Landau damping by l carriers (the horizontal hatching) and by h carriers (the vertical hatching) in the superconducting state with a gap 2Δ in the spectrum of l carriers. (It is assumed that the h carriers are in a normal, gapless state at $T<T_c$.)

$\gamma_q=(\sqrt{l/\pi})2\omega_q^2/qv_{Fl}$, since its phase velocity u is lower than the Fermi velocity of the l carriers, v_{Fl} (but higher than the Fermi velocity of the h carriers, v_{Fh}). In the transition to the superconducting state as a result of the opening of a gap in the spectrum of quasiparticles (l carriers), the Landau damping at energies $\omega<2\Delta(T)$ quickly comes to an end as T is lowered. In this case the damping rate is given by the following expression according to the BCS model:

$$\gamma_l(\omega_q,T)=\omega_q \frac{u}{v_{Fl}g}(\omega_q,T). \tag{1}$$

A general expression for the function g was given in Ref. 2. At low frequencies, $\omega\ll\Delta(T)$, that expression becomes[1]

$$g(T)=\sqrt{\pi}[\exp\{\Delta(T)/T\}+1]^{-1}. \tag{2}$$

Let us assume that the damping of acoustic plasmons due to elastic scattering of carriers by lattice defects and impurities (Drude damping), with a time scale τ_D, is slight. In this case long-wave acoustic plasmons with frequencies in the transparency region (Fig. 1),

$$\tau_D^{-1}\lesssim\omega_q\approx qu\lesssim 2\Delta(T), \tag{3}$$

may make a significant contribution to the thermal conductivity of the crystal, as do acoustic phonons.

3. Let us examine the thermal conductivity of a Bose gas of slightly damped acoustic plasmons inside the gap. By analogy with Ref. 2, we write the Boltzmann kinetic equation for a small nonequilibrium increment $N_1(q,T)$, due to a temperature gradient, in the equilibrium Bose–Einstein distribution function $N_0(q,T)=[\exp(\omega_q/T)]^{-1}$ of the thermal acoustic plasmons. It follows that we have

$$N_1(q,T)=-\mathbf{u}\nabla T\tau_{\rm pl}(\omega_q,T)\frac{\partial N_0}{\partial T}, \tag{4}$$

where

$$\tau_{\rm pl}^{-1}(\omega_q,T)=\tau_D^{-1}+\gamma_l(\omega_q,T). \tag{5}$$

The corresponding component of the heat flux $\mathbf{Q}=-\kappa\nabla T$ due to nonequilibrium acoustic plasmons is given by the following expression, by analogy with the acoustic-phonon component:

$$\mathbf{Q}_{\rm pl}\equiv\int\frac{d^3q}{(2\pi)^3}\mathbf{u}\omega_q N_1(q,T)=-\frac{\nabla T}{8\pi T^2 d}\int_{\tau_D^{-1}}^{\infty}\frac{\omega^2\tau_{\rm pl}(\omega,T)e^{\omega/T}}{(e^{\omega/T}-1)^2}\,d\omega, \tag{6}$$

where d is the lattice constant along the **c** axis.

The derivation of (6) reflects the circumstance that in a layered crystal with a 2D band spectrum the frequency of acoustic plasmons is a weak function[23] of the transverse component of the momentum **q** ("transverse" here means perpendicular to the layers), so it is quite accurate to assume that the velocity vector **u** lies in the plane of the layers. Because of this circumstance, an anomalous increment in the thermal conductivity arises in this model only for a gradient of T in the $a-b$ plane. There is no such increment if ∇T is directed along the **c** axis. This conclusion agrees with experimental data.[3,4]

In $YBa_2Cu_3O_{7-\delta}$ single crystals without twinning, one observes an additional anisotropy of the thermal conductivity in the $a-b$ plane, in both the normal and superconducting states. This additional anisotropy is due to an ordering of 1D CuO chains along the **b** axis in the $z=0$ basal planes, so the relation $\kappa_b>\kappa_a$ holds.[6] This anisotropy is characteristic of both the electron and phonon components of the thermal conductivity, as can be seen from the nonmonotonic behavior of the difference $(\kappa_b-\kappa_a)$ below T_c. For the plasmon component in (6), the anisotropy of the thermal conductivity in the $a-b$ plane may be due to an anisotropy of the velocity **u** and of the gap parameter Δ associated with the 1D chains. For simplicity, however, we will ignore this point below and use the model of a layered crystal with a period d.

As a result, the increment in the thermal conductivity in the plane of the layer due to the 2D Bose gas of thermal acoustic plasmons is

$$\kappa_{\rm pl}(t)=\frac{T_c}{8\pi d}f(t),\quad f(t)=\frac{t^2}{\nu}\int_{\nu/t}^{\infty}\frac{x^3e^x dx}{[1+(\beta/\nu)txg(x)](e^x-1)^2}, \tag{7}$$

where

$$t=T/T_c,\quad \nu=1/\tau_D T_c,\quad \beta=u/v_{Fl}. \tag{8}$$

It follows from (7) that in the region $(T_c-T)\ll T_c$ the plasmon thermal conductivity increases with decreasing T in proportion to $\Delta^2(T)\sim(T_c-T)$, in contrast with the bipolaron thermal conductivity,[10] which increases as $(T_c-T)^{3/2}$ at $T<T_c$. At low tempera-

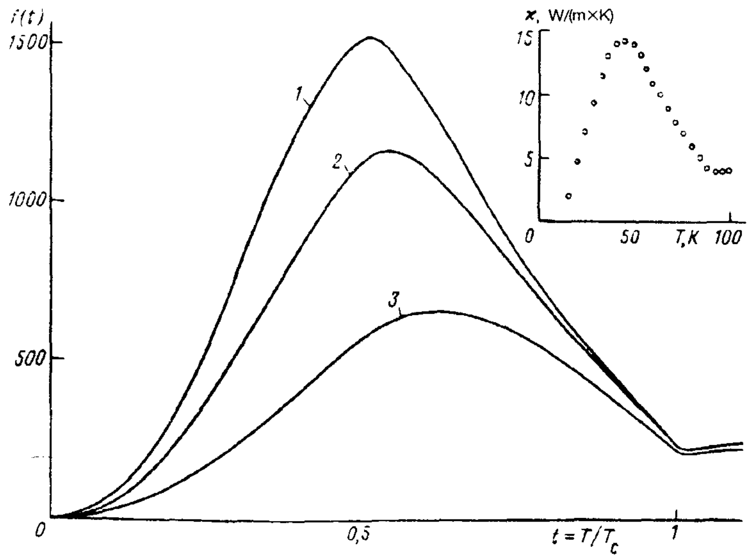

FIG. 2. The function $f(t)$ for $\beta=0.1$ and various values of the parameter ν. 1) $\nu=0.8\times10^{-3}$; 2) 1.2×10^{-3}; 3) 2.7×10^{-3}.

tures $T \ll T_c$, where the relation $\Delta(T) \gg T$ holds, the T dependence of the plasmon thermal conductivity is quadratic, $\kappa_{\rm pl} \sim T^2$. This behavior is again different from that of the thermal conductivity of a Bose gas of bipolarons:[10] $\kappa_{\rm BP} \sim T^3$.

Figure 2 shows the functional dependence $f(t)$ for various values of the parameter ν under the condition $\beta = \text{const}$. We see a satisfactory qualitative agreement between the theoretical curves and the experimental T dependence of the thermal conductivity in $YBa_2Cu_3O_{7-\delta}$ (after the subtraction of the phonon component), as shown in the inset in Fig. 2. The agreement improves when the l-carrier component of the thermal conductivity is taken into account (in the BCS theory[25]).

The increase in the maximum on the $f(t)$ curve with decreasing ν (i.e., with increasing τ_D) can explain the experimentally observed increase in the maximum of the thermal conductivity in $YBa_2Cu_3O_{7-\delta}$, when samples are annealed in oxygen,[5] if we assume that the Drude damping of the acoustic plasmons in the superconducting state is due primarily to a scattering of the h carriers by charged oxygen vacancies, whose concentration falls off as the annealing proceeds. The time scale of this scattering may be far longer than the transport scattering time of the l carriers.

At the same time, the suppression of the maximum of the thermal conductivity by a magnetic field below T_c, which was observed in Ref. 26, may be due to an increase in the Landau damping of acoustic plasmons by l carriers inside the normal cores of Abrikosov vortices.

Finally, the reason why there is no phonon-related peak in the thermal conductivity along the **c** axis may be that the spectrum of acoustic phonons lies in a region of strong

Landau damping by h carriers (the vertically hatched region in Fig. 1) which remain nondegenerate below T_c and thus do not go into a superconducting state down to $T \ll T_c$.

4. The maximum in the thermal conductivity below T_c in the high-T_c superconductors[3–9] based on p-type layered cuprate compounds with anomalously narrow 2D bands near the Fermi level[18] may thus arise because a neutral Bose gas of weakly damped thermal acoustic plasmons inside the gap $2\Delta(T)$ contributes to the thermal conductivity. The existence of a branch of acoustic plasmons in such cuprates, along with high-frequency polar optical phonons (oxygen vibrational modes), promotes an intensification of the electron–electron attraction and an increase in T_c (Refs. 12 and 27) by virtue of a plasmon mechanism for high-T_c superconductivity.

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