A simple way to estimate the value of $\bar{\alpha}\equiv\alpha(m_Z^3)$

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To obtain the value of the electromagnetic coupling constant at $q^2=m_Z^2$, $\bar{\alpha}$, which plays the key role in electroweak physics, we must integrate the cross section of e^+e^--annihilation into hadrons divided by $(s-m_Z^2)$ over s from the threshold to infinity. By combining, for each flavor channel, the contribution of the lowest resonance with the perturbative QCD continuum, we obtain $1/\bar{\alpha}=128.89\pm0.06$. This value is close to the known result which was obtained with purely experimental inputs, $1/\bar{\alpha}=128.87\pm0.12$. © *1994 American Institute of Physics.*

A detailed analysis of the electroweak observables starts from three input parameters: G_μ, the Fermi coupling constant (extracted from muon decay), m_Z, the Z boson mass (measured at LEP), and $\bar{\alpha}$, the electromagnetic coupling constant at $q^2=m_Z^2$, obtained from dispersion relations. In fact, a Born approximation to the minimal standard model which starts with $\bar{\alpha}$ [rather than $\alpha\equiv\alpha(0)=1/137.0359895(61)$] reproduces the precise experimental values of the Z decay parameters (obtained at LEP) and of the W mass (obtained at hadron colliders) with unexpectedly high accuracy.[1,2] For example, for the ratio of the vector and the axial coupling constants of the Z boson to the charged leptons, we obtain the following value in the $\bar{\alpha}$ Born approximation:[2]

$$[g_V/g_A]_{\bar{\alpha}}=0.0753(12), \tag{1}$$

and the latest experimental numbers are[3]

$$[g_V/g_A]_{\rm LEP}=0.0711(20), \tag{2}$$

$$[g_V/g_A]_{\rm LEP+SLD}=0.0737(18). \tag{3}$$

If instead of $\bar{\alpha}$ we use $\alpha(0)$, we then get

$$[g_V/g_A]_\alpha=0.152,$$

which is about 40σ's away from the experiment, as was pointed out in Ref. 2. The value of $\bar{\alpha}$ is of fundamental importance, and its error determines the uncertainty in the theoretical prediction (1).

The quantity $\bar{\alpha}$ is defined in terms of the formulas

$$\bar{\alpha}=\frac{\alpha}{1-\delta\alpha}, \tag{4}$$

$$\delta\alpha=\Sigma'_{\gamma}(0)-\frac{\Sigma_{\gamma}(m_Z^2)}{m_Z^2}, \tag{5}$$

where the charge leptons and the five quark flavor contributions to the photon polarization operator should be taken into account in (5). The contributions of $(t\bar{t})$ and $(W\bar{W})$ loops may be omitted in (5); these numerically small contribution usually are attributed to proper, electroweak, radiative corrections.[4] The following integral representation for $\delta\alpha$ is valid:

$$\delta\alpha=\frac{m_Z^2}{4\pi^2\alpha}\int\frac{\sigma_{e^+e^-\to\text{all}}(s)}{m_Z^2-s}\,ds, \tag{6}$$

where the integral goes from the threshold to infinity and its principal value at $s=m_Z^2$ should be used. The lepton contribution of e, μ, and τ to (6) can be easily calculated, and we obtain

$$(\delta\alpha)_l=\frac{\alpha}{3\pi}\left[\Sigma\ln\frac{m_Z^2}{m_l^2}-\frac{5}{3}\right]=\frac{\alpha}{3\pi}[22.5+11.8+6.2]=0.0314. \tag{7}$$

For the hadronic contribution in Ref. 5 the following value was obtained (see also Ref. 6):

$$(\delta\alpha)_h=0.0282(9). \tag{8}$$

To obtain this value, the experimental cross section for e^+e^--annihilation into hadrons below $s_0=(40\ \text{GeV})^2$ and the result of the parton model given above, s_0, were used in Refs. 5 and 6.

The difficulty in the theoretical determination of $(\delta\alpha)_h$ comes from its logarithmic dependence on the infrared cutoff. It was mentioned in Ref. 7 that the result of the dispersion calculation of $(\delta\alpha)_h$ can be reproduced by using perturbative QCD with the effective "quark masses"

$$\begin{aligned}&m_u=53\ \text{MeV},\quad m_d=71\ \text{MeV},\quad m_s=174\ \text{MeV},\\&m_c=1.5\ \text{GeV},\quad m_b=4.5\ \text{GeV}.\end{aligned} \tag{9}$$

Unfortunately, we cannot attribute any physical meaning to these values of m_u and m_d.

Our aim here is to present a simple sensible model for $\sigma_{e^+e^-\to\text{hadrons}}$, which can simulate the result given in (8). To do this we use one physical resonance (ρ, ω, φ, J/ψ, and Υ) at the beginning of the spectrum and then starting from $E_i=m_i+(\Gamma_i/2)$, the improved QCD parton model continuum in each quark channel.

For the resonance contribution we use the Breit–Wigner formula

$$\sigma_{ee}=\frac{3\pi\Gamma_{ee}\Gamma}{E^2[(E-m)^2+\Gamma^2/4]}. \tag{10}$$

Substituting it in (6), ignoring terms of the order of $(m/m_Z)^2$, and integrating from $-\infty$ to $m+(\Gamma/2)$, we obtain

$$(\delta\alpha)_{\text{resonance}}=\frac{3}{\alpha}\frac{\Gamma_{ee}}{m}\frac{3}{4}.$$

Thus the vector meson contributions to $\delta\alpha$ are

$$\begin{array}{cccccc} & \rho & \omega & \varphi & J/\psi & \Upsilon \\ \delta\alpha & 0.00274(13) & 0.00024 & 0.00042 & 0.00053 & 0.000045. \end{array} \tag{12}$$

Here we take into account the experimental uncertainty for ρ-meson contribution as the only noticeable uncertainty.

For continuum contribution we use the following formulas:

$$\sigma_{I=1}=2\pi\frac{\alpha^2}{s}\left(1+\frac{\alpha_s(s)}{\pi}\right), \tag{13}$$

$$\sigma_{I=0}=\frac{2\pi}{9}\frac{\alpha^2}{s}\left(1+\frac{\alpha_s(s)}{\pi}\right), \tag{14}$$

$$\sigma_{s\bar{s}}=\frac{4\pi}{9}\frac{\alpha^2}{s}\left(1+\frac{\alpha_s(s)}{\pi}\right), \tag{15}$$

$$\sigma_{c\bar{c}}=\frac{16\pi}{9}\frac{\alpha^2}{s}\sqrt{1-\frac{4m_c^2}{s}}\left(1+\frac{2m_c^2}{s}\right)\left(1+\frac{\alpha_s(s)}{s}\right), \tag{16}$$

$$\sigma_{b\bar{b}}=\frac{4\pi}{9}\frac{\alpha^2}{s}\sqrt{1-\frac{4m_b^2}{s}}\left(1+\frac{2m_b^2}{s}\right)\left(1+\frac{\alpha_s(s)}{\pi}\right), \tag{17}$$

and we use for $\alpha_s(s)$ the formula

$$\alpha_s(s)=\frac{12\pi}{(33-2n_f)\ln s/\Lambda^{(n_f)^2}} \tag{18}$$

with $\alpha_s(m_Z=0.129(5)$ as the input [this one-loop value corresponds to 0.125(5) at the three loops; this value is extracted from the latest LEP data[2]]. We assume $n_f=5$ for $s>m_\Upsilon^2$, $n_f=4$ for $m_\Upsilon^2>s>m_{J/\psi}^2$, $n_f=3$ for $m_{J/\psi}^2>s<m_\varphi^2$, and $n_f=2$ for $m_\varphi^2>s>(m_p+\Gamma_\rho/2)^2$. This corresponds to $\Lambda^{(5)}=160$ MeV, $\Lambda^{(4)}=220$ MeV, $\Lambda^{(3)}=270$ MeV, and $\Lambda^{(2)}=300$ MeV.

Substituting (13)–(18) into (6) with $m_c=m_b=0$, we obtain

$$\begin{array}{cccccc} & I=1 & I=0 & s\bar{s} & c\bar{c} & b\bar{b} \\ \delta\alpha & 0.01174 & 0.00133 & 0.00249 & 0.00741 & 0.00123. \end{array} \tag{19}$$

Summing up contributions from (12) and (19), we find

$$(\delta\alpha)_h=0.0282, \quad \bar{\alpha}=(128.87)^{-1}. \tag{20}$$

Comparing results (8) with those obtained by integrating the experimental data ($\delta\alpha_h=0.0282$ and $\bar{\alpha}=[128.87(12)]^{-1}$, we see that the agreement is astonishing. The contribution of α_s correction in (19) is rather small, 0.0087+0.00010+0.00018+0.00042 +0.00006=0.00163, so even if the light gluino octet slows down α_s running in order to accommodate the α_s values measured during quarkonium decays,[8] $(\delta\alpha)_h$ will decrease only by 0.0002.

A few comments are in order.

1. Taking the contributions $\sim\alpha_s^2$ in the continuum cross section into account and using the next-to-leading order formula for $\alpha_s(s)$, we increase $(\delta\alpha)_h$ by 0.00045; negative contribution of the term $\sim\alpha_s^3$ appears to be approximately twice as large. In the beauty and the charm channels the third loop gives a much smaller contribution than the second loop (numerically both are negligible), so we can trust the continuum calculation. In the strange channel the third loop contribution equals that of the second, while in the $I=1$ and $I=0$ channels it is twice as large. Below, say, 1.5 GeV the perturbative continuum therefore cannot be accepted. Allowing the physical continuum variation at the level of $\pm 15\%$ near the three-plus-one loop perturbative continuum value in the region 1–2 GeV, we obtain a variation of ± 0.0004 in $(\delta\alpha)_h$.

2. Experimental uncertainty in Γ_{ll} of the vector resonances lead to a $(\delta\alpha)_h$ variation of the order of 0.0002, while that in $\alpha_s(m_Z)$ lead to a variation of 0.0001. Both variations are small compared with the uncertainty 0.0009 in (8).

3. Subtracting from the ρ contribution the integral over the Breit–Wigner formula from $-\infty$ to two-pion threshold, we decrease it by

$$\delta\alpha_{\text{sub}}=\frac{3\Gamma_{ee}}{2\pi\alpha m_\rho}\,2\arctan\Gamma_\rho/[2(m_\rho-2m_\pi)]=0.00017. \tag{21}$$

4. Taking into account the heavy quark masses $m_c=1.6$ GeV and $m_b=4.7$ GeV, we decrease $(\delta\alpha)_h$ correspondingly by

$$(\delta\alpha_h)_m=0.00031+0.00008=0.00039. \tag{22}$$

5. Finally, at energies $E=m_i+(\Gamma_i/2)$ our model curve for $\sigma_{e^+e^-\to\text{hadrons}}$ is discontinuous. To understand the $(\delta\alpha)_h$ sensitivity for the details of the model, we change it in the following way: we continue ρ, ω, φ, and J/ψ resonance curves up to their intersection with the quark continuum. In this way $(\delta\alpha)_h$ increases,

$$\delta(\delta\alpha)_h=0.00051. \tag{23}$$

Subtracting from (23) the sum of (22) and (21) and taking the uncertainty from point (1) for the total shift, we obtain

$$(\delta\alpha)_h=0.0281(4), \quad \bar{\alpha}=[128.89(6)]^{-1}. \tag{24}$$

It is evident, therefore, that the value of $(\delta\alpha)_h$ is insensitive to the particular features of the model for $\sigma_{e^+e^-\to\text{hadrons}}$. A more refined model, which takes into account all known resonances in each flavor channel, gives[9] $(\delta\alpha)_h=0.0275(2)$.

For a real progress in reducing the error in (8) it is necessary to improve the systematic error in the cross section for the e^+e^- annihilation into hadrons in the background region below 3 GeV.[5,10]

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