Is there a dramatic suppression of $n\bar{n}$ transitions in a medium?

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It is shown that $n\bar{n}$ transitions in nuclear matter are suppressed by a factor of only 0.5 in comparison with oscillations in vacuum. The limitation $\tau_{n\bar{n}}>3\times10^{31}$ yr on the period of $n\bar{n}$ oscillations in vacuum follows. This limitation is 31 orders of magnitude stronger than existing limitations. © *1994 American Institute of Physics.*

Calculations on $n\bar{n}$ transitions in a medium are based on a potential model (Refs. 1–7, for example) which can be summarized as follows: The neutron is in a self-consistent potential U_n. An optical potential $U_{\bar{n}}$ corresponds to the antineutron. The energy gap $\delta U=U_{\bar{n}}-U_n$ leads to a very strong suppression of $n\bar{n}$ transitions. The reasons for the dramatic suppression of the process are not clear from the standpoint of the microscopic model (dynamic $n\bar{n}$ transition,[8] annihilation). At any rate, it is worthwhile to carry out a calculation not based on a potential model. We offer such a calculation below.

We consider $n\bar{n}$ transitions in nuclear matter. We incorporate the potential U_n in the neutron wave function $n(x)$:

$$n(x)=\frac{1}{\sqrt{V}}\,e^{-i\epsilon_n t+i\mathbf{p}_n\cdot\mathbf{x}},\qquad \epsilon_n=\mathbf{p}_n^2/2m+U_n. \tag{1}$$

The Hamiltonian of the interaction is

$$H_I(t)=H(t)+H_{n\bar{n}}(t). \tag{2}$$

Here $H_{n\bar{n}}$ is the Hamiltonian of the oscillations:

$$H_{n\bar{n}}(t)=\epsilon\int d^3x(\bar{\Psi}_{\bar{n}}\Psi_n+\bar{\Psi}_n\Psi_{\bar{n}}), \tag{3}$$

where Ψ_n and $\Psi_{\bar{n}}$ are the fields of the neutron and the antineutron, respectively, and $\epsilon=(m_2-m_1)/2$, where $m_{1,2}$ are the masses of stationary states. We take the Hamiltonian H in the general form $H(t)=(\text{all }\bar{n}\text{–medium interactions})-U_n$.

Figure 1 shows the $n\bar{n}$ transition, annihilation. The amplitude for the process is singular:

$$\epsilon\,\frac{1}{\epsilon_n-\mathbf{p}_n^2/2m-U_n}\,M\sim\frac{1}{0}. \tag{4}$$

This is an infrared divergence, which stems from the zero momentum transfer at the vertex of the $n\bar{n}$ transition. There is no mechanism of cancellation by radiation correc-

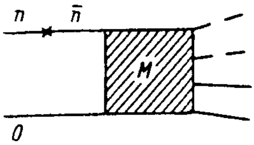

FIG. 1. $n\bar{n}$ transition, annihilation in nuclear matter.

tions. One way to eliminate the divergences might be to examine the problem on a finite time interval $(t,0)$. The problem has a second distinctive feature: As will become clear below, we need to find the exact dependence on t. Accordingly, a model of the type in Fig. 1 is insufficient. On the interval $(t,0)$ we need to evaluate a matrix element, retaining all the terms of a formal expansion of the operator $T\exp\{-i\int_0^t dt_1 H(t_1)\}$. In lowest order in $H_{n\bar{n}}$, the U matrix is

$$\langle 0n_p|\hat{U}(t,0)-I|n_p0\rangle = iT_{ii}(t,0) = i[T_{ii}^a(t,0)+T_{ii}^b(t,0)], \tag{5}$$

$$iT_{ii}^a(t,0) = \left\langle n_p \left| (-i)^2 \int_0^t dt_\alpha \int_0^{t_\alpha} dt_\beta H_{n\bar{n}}(t_\alpha) H_{n\bar{n}}(t_\beta) \right| n_p \right\rangle, \tag{6}$$

$$iT_{ii}^b(t,0) = \sum_{k=1}^{\infty} \left\langle 0n_p \left| (-i)^{k+2} \int_0^t dt_\alpha \int_0^{t_\alpha} dt_1 \ldots \int_0^{t_{k-1}} dt_k \int_0^{t_k} dt_\beta H_{n\bar{n}}(t_\alpha) H(t_1) \ldots H(t_k) H_{n\bar{n}}(t_\beta) \right| 0n_p \right\rangle. \tag{7}$$

Here $|0n_p\rangle$ is a state vector of a medium which contains a neutron with a 4-momentum $p=(\mathbf{p}_n^2/2m+U_n,\ \mathbf{p}_n)$. Expressions (6) and (7) correspond to parts a and b of Fig. 2. The T block is expanded in Fig. 3. If we write H in terms of scattering and annihilation Hamiltonians, $H=H_a+H_s$, we have $\|H_a\| \gg \|H_s\|$. To emphasize this circumstance, we group the annihilation diagrams in the first sum (Figs. 3b, 3c, ...). In an S-matrix formulation of the problem we have $(t,0)\to(\infty,-\infty)$, and expressions (6) and (7) contain singular antineutron propagators.

We take the Green's function in the form

$$G(x-x_1) = -i\int \frac{d^3p_n}{(2\pi)^3} e^{i\mathbf{p}_n\cdot(\mathbf{x}-\mathbf{x}_1)-i(t-t_1)(\mathbf{p}_n^2/2m+U_n)} \theta(t-t_1). \tag{8}$$

For Fig. 2a we have

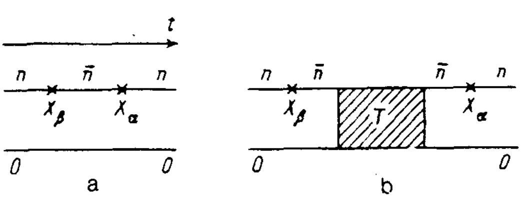

FIG. 2. $n\bar{n}$ transitions in a medium.

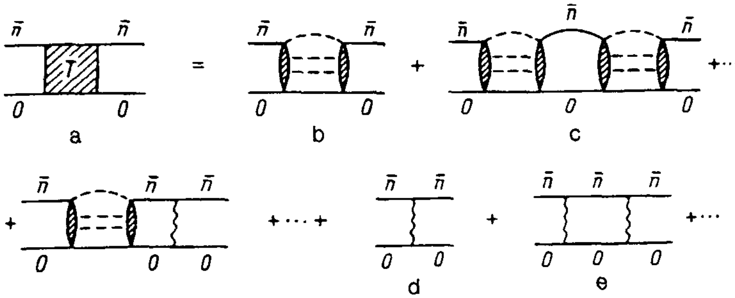

FIG. 3. Diagrams for the decay of an $\bar{n}$-nucleus state.

$$iT^a_{ii}(t,0)=(-i)^2\int_0^t dt_\alpha\int_0^{t_\alpha}dt_\beta\int d^3x_\alpha d^3x_\beta n_p^*(x_\alpha)\epsilon iG(x_\alpha-x_\beta)\epsilon n_p(x_\beta)$$
$$=-\epsilon^2t^2/2. \tag{9}$$

Although this expression follows from the formal definition of the Green's function, we can also calculate multitails from (8). We have found a natural regularization. In the limit $t\to\infty$, expression (9) diverges in the same way as the square modulus of (4).

The matrix element T^b_{ii} can be expressed in terms of the matrix element $T^{\bar{n}}_{ii}$, corresponding to the decay of an $\bar{n}$ nucleus (Fig. 3):

$$iT^{\bar{n}}_{ii}(t_\alpha,t_\beta)=\sum_{k=1}^{\infty}(-i)^k\left\langle 0\bar{n}_p\left|\int_{t_\beta}^{t_\alpha}dt_1...\int_{t_\beta}^{t_{k-1}}dt_kH(t_1)..H(t_k)\right|0\bar{n}_p\right\rangle, \tag{10}$$

where $|0\bar{n}_p\rangle$ is a state vector of a medium containing an antineutron with a 4-momentum equal to the 4-momentum of the original neutron. Singling out the Green's functions of the antineutron between the T block and the ϵ vertices, and using the relation

$$\int d^3x_\beta\langle T[\Psi_{\bar{n}}(x_k)\Psi^+_{\bar{n}}(x_\beta)]\rangle n(x_\beta)=\bar{n}(x_k) \tag{11}$$

(Schrödinger fields), we find

$$iT^b_{ii}(t,0)=\epsilon^2\sum_{k=1}^{\infty}(-i)^{k+2}\int_0^t dt_\alpha\int_0^{t_\alpha}dt_1...\int_0^{t_{k-1}}dt_k\int_0^{t_k}dt_\beta\langle 0\bar{n}_p|H(t_1)...H(t_k)|0\bar{n}_p\rangle. \tag{12}$$

Using the formula

$$\int_a^b dx\int_a^x dyf(x,y)=\int_a^b dy\int_y^b dxf(x,y), \tag{13}$$

we sequentially change the order of integration. We find

$$T_{ii}(t,0)=i\epsilon^2t^2/2-\epsilon^2\int_0^t dt_\alpha\int_0^{t_\alpha}dt_\beta T_{ii}^{\bar{n}}(t_\alpha,t_\beta). \tag{14}$$

Equating the imaginary parts and using the unitarity condition $UU^+=1$, which signifies conservation of the probability on the interval $(t,0)$, we find

$$W(t)=\epsilon^2t^2-\epsilon^2\int_0^t dt_\alpha\int_0^{t_\alpha}dt_\beta W_{\bar{n}}(t_\alpha,t_\beta). \tag{15}$$

Here $W(t)$ is the probability for the overall process, and $W_{\bar{n}}(t_\alpha,t_\beta)$ is the probability for the decay of the state of the $\bar{n}$ medium over a time $\tau=t_\alpha-t_\beta$ ($\tau=0$ is the time at which the state forms). The two-step process has thus been reduced to the decay of an $\bar{n}$ nucleus which satisfies the exponential decay law

$$W_{\bar{n}}(t_\alpha,t_\beta)=1-e^{-\Gamma(t_\alpha-t_\beta)}, \tag{16}$$

where $\Gamma\sim100$ MeV is the annihilation width of the $\bar{n}$ nucleus. We finally find

$$W(t)=\epsilon^2t^2\left[\frac{1}{2}+\frac{1}{\Gamma t}+\frac{1}{\Gamma^2t^2}(e^{-\Gamma t}-1)\right]. \tag{17}$$

In the limit $\Gamma t\ll1$ (the low-density limit) we have $W(t)=\epsilon^2t^2$. This result agrees with the probability for the $n\bar{n}$ transition in vacuum. We are interested in the strong-absorption regime, $\Gamma t\gg1$. We have $W(t)=\epsilon^2t^2/2$. The suppression factor in a medium is 1/2. For scattering by a black nucleus, a factor of 1/2 arises again: $\sigma_{\text{abs}}=\sigma_{\text{tot}}/2$. This result was to be expected, although no assumptions were made in the derivation of (15) from (5). The $\bar{n}$ nucleus arises automatically. Decay law (16) is also noncontroversial.

A limitation on the period of $n\bar{n}$ oscillations in vacuum, $\tau_{n\bar{n}}=1/\epsilon$, follows from the condition $W(T)<1$, where T is the limitation on the annihilation lifetime of nuclei. For $T=4.3\times10^{31}$ (Ref. 9) we find

$$\tau_{n\bar{n}}>T/\sqrt{2}=3\times10^{31}\ \text{yr}. \tag{18}$$

The corresponding theoretical and experimental limitations are, respectively, $\tau_{n\bar{n}}>1$ yr (Refs. 1–7 and 10) and $\tau_{n\bar{n}}>10^7$ s (Ref. 11). Let us briefly discuss the reasons for the discrepancy with the potential model:

$$(i\partial_t+\nabla^2/2m-U_n)n(x)=\epsilon\bar{n}(x),$$
$$(i\partial_t+\nabla^2/2m-U_{\bar{n}})\bar{n}(x)=\epsilon n(x). \tag{19}$$

With $U_n=\text{const}$ and $U_{\bar{n}}=\text{const}$, and in lowest order in ϵ, the probability for the process found from (19) is

$$W_{\text{pot}}(t)=2\,\text{Im}\,i(\epsilon/\delta U)^2[1-i\delta Ut-\exp(-i\delta Ut)]. \tag{20}$$

At the same time, we consider only diagrams in which there is a rescattering of the $\bar{n}$ through a zero angle (Figs. 3d, e, ...). Working from expressions (5)–(7), we can explicitly evaluate the T block, using standard approximations: the impulse approximation and the approximation of a frozen medium. We also use the unitarity condition (although $U_{\bar{n}}$ is probably "anti-Hermitian"). As a result, we find (20). Consequently, 1) an approach with finite times has been tested in the case of an exactly solvable model, and 2)

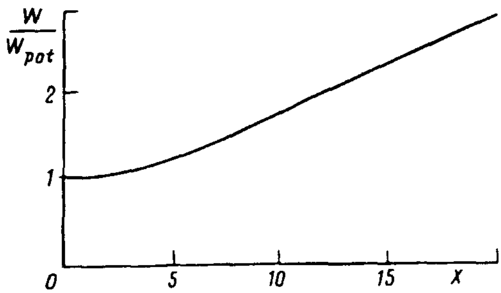

FIG. 4. Ratio of the probability for an $n\bar{n}$ transition in a medium derived in the present letter [expression (17)] to the probability for the process derived with the help of the potential model [expression (20)] with $\delta U = -i\Gamma/2$.

the potential model implies, from the standpoint of the microscopic theory, an infinitely repeated rescattering of the $\bar{n}$ through a zero angle. (This fact is known in the standard S-matrix theory of multiple scattering.)

A scheme of this sort can clearly be satisfactory in problems in which there is a weak absorption: $x = \Gamma t < 1$. Specifically, it can be seen from Fig. 4 that the potential model is valid in the region $x < (5-10)$. In experiments on $K^0\bar{K}^0$ and $\nu_\alpha \nu_\beta$ oscillations, the condition $\Gamma t = \rho \sigma_t v t \ll 1$ holds. In the overwhelming majority of problems which use an $\bar{N}$-nucleus optical potential, the parameter Γt or its analog does not exceed 10. In the problem under discussion here, we are dealing with the regime of limiting absorption, $\Gamma t \to \infty$.

The following assertions have also been proved: 1. The corrections to the model (these are processes which cannot be incorporated in T or U_n) can be ignored. 2. The results reported above also hold for a generalized nuclear model. 3. We have considered conditions under which limitation (18) is "spoiled." For this purpose, we studied the relationship between the S-matrix theory and an approach with finite times for diagrams of various types. A strong suppression is possible if the vertex of the $n\bar{n}$ transition is a 3-tail (i.e., if $q \neq 0$). This situation implies another oscillation Hamiltonian in vacuum. These and other results will be published in a separate paper.

If limitation (18) seems too radical, the material presented above can be looked at from the following standpoint. The reason for the suppression of the process was an energy gap. It has been shown that this factor is a price paid for using the potential model. That model is not correct in the case $\Gamma t \gg 1$. Do other suppression mechanisms exist? This question will be the subject of further research. We do not believe that there are such mechanisms (see the preceding paragraph).

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