Resonance effects in the high-frequency conductance of a 1D quantum channel in the ballistic regime

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This letter analyzes the dynamic conductance of a quantum ballistic channel connecting two highly conducting regions across which an alternating voltage is applied. The frequency dependence of the admittance may be oscillatory, because of spatial resonances of charge-density waves that propagate along the channel. © *1994 American Institute of Physics.*

Time-varying processes in the terahertz range in quantum structures have obviously attracted increased interest in recent years.[1-5] To the best of our knowledge, however, there has been no study of the dynamic conductance of quantum channels in the ballistic regime, although there have been a fairly large number of studies of the steady-state conductance,[6] the steady-state current induced by an alternating perturbation,[5,7-9] and time-varying tunneling.[10-13] In the present letter we show that resonance effects arise in the dynamic conductance of quantum channels because of the propagation of charge-density waves along a channel.

The dynamic conductance (or admittance) of a quantum channel is defined as the ratio of the complex amplitude of the alternating current in the external circuit to the amplitude of the alternating voltage. A distinctive feature of a calculation of the dynamic conductance is that the current in the external circuit is by no means equal to the quantum-mechanical transport current, since under time-varying conditions the transport current depends on not only the time, but also the coordinate along the channel. As will be shown below, the time-varying current in the external circuit is an integral function of the transport current over the entire length of the channel.

Let us consider a quantum channel of length L which connects two highly conducting regions (electrodes), across which an alternating voltage $V_1\cos(\Omega t)$ is applied. We assume that the conductance of the electrodes is so high that the characteristic frequencies of electronic processes (plasma frequencies and the Maxwellian relaxation frequency) in the electrodes are high in comparison with the characteristic frequencies of electron transport in the channel. Since we are interested in fairly short channels, we also ignore retardation effects. The necessary condition here ($L \ll c/\Omega$) holds well for ballistic channels in the terahertz range. Under these conditions, the surfaces of the electrodes can be assumed to be equipotentials. We describe the distribution of the alternating potential in the channel by means of a function $\phi(x)$, which is determined by the electrode geometry and which we assume to be a given. We normalize the function $\phi(x)$ in such a way that the conditions $\phi(0)=0$ and $\phi(L)=1$ hold. The alternating potential is then

$$V_{ac}(x,t)=V_1\phi(x)\cos(\Omega t).$$

As we will see below, the function $\phi(x)$ plays an important role in the frequency dependence of the admittance.

We know that the application of a periodic perturbation results in the emission and absorption of photons $\hbar\Omega$ by electrons. As a further result, side bands (quasienergies) $\epsilon+n\hbar\Omega$ (n is an integer) appear in the spectrum of electron waves. In this letter we are concerned with the high-frequency limit $\hbar\Omega\rightarrow eV_1$. In this limit, the absorption or emission of simply a single photon may prove important.[1] The current response at the frequency Ω is governed by an interference of electron waves corresponding to different bands. In the course of this interference, a charge density $\rho(x,t)$ arises. This charge density varies with the time and is nonuniform along the channel. In the case of interest here, that of above-barrier transmission, this charge density takes the form of traveling charge-density waves. By virtue of the continuity equation, the quantum transport current $j(x,t)$ also depends on the time and the coordinate along the channel. There is thus the question of how we are to determine the alternating current in the external circuit. Clearly, the total current is equal to the sum of the transport current and the displacement current; specifically, this sum is constant in all cross sections between the surfaces of the electrodes. The displacement current has two components. One of them is $C\times dV_1/dt$, where C is the mutual capacitance of the electrodes. The other component is associated with charges moving in the gap between the electrodes. The charge-density wave in the channel gives rise to an alternating electric field in the surrounding volume and thus to a displacement current. The most convenient way to calculate this component of the total current is as dQ_1/dt, where Q_1 is the charge induced at one of the electrodes (for definiteness, the electrode on the left) by the charges moving in the channel. The total current in the external circuit can thus be written

$$J(t)=j(x=0,t)+dQ_1/dt+C\times dV_1/dt,$$

where the first term is the transport current across the boundary with the left electrode. The charge Q_1 can be expressed in terms of $\rho(x,t)$ and the Green's function of the Laplace equation for the potential in the volume between the electrodes. Making use of the circumstance that $\rho(x,t)$ is related to the current $j(x,t)$ by the continuity equation, we can then express Q_1 in terms of the current and the function $\phi(x)$ introduced above. As a result, we find the following expression for the current in the external circuit:

$$J(t)=\int_0^L dx\times j(x,t)\times d\phi/dx+C\times dV_1/dt. \tag{1}$$

To find the current in the external circuit, we need to calculate the quantum-mechanical transport current. This current is found from the solution of the Schrödinger equation with the time-dependent potential $V_{ac}(xt)$. To calculate the current $J(t)$, we must therefore use the potential $\phi(x)$ twice: once in the electrostatic part of the problem, to find the displacement current, and again in the quantum-mechanical part, to find the transport current. The Schrödinger equation generally includes, along with the alternating potential, a static potential $V_0(x)$. The latter can be used to model the change in the kinetic energy of the electrons as they move out of the electrodes into the channel. The existence of a static potential is of fundamental importance if the Fermi energy ϵ_F is smaller than $\max[V_0(x)]$. If the condition $\epsilon_F>\max[V_0(x)]$ holds instead, then the static potential is unimportant for the effects discussed in this letter. Below we write the solution for the case $V_0=0$, for an arbitrary distribution of the time-varying potential along the channel. This solution was derived in the Born approximation. In calculating the current we need to use two solutions: one for the waves incident on the channel from the left electrode, and one for the waves incident from the right electrode. The first of these solutions is

$$\psi_>(x,t)=e^{i(kx-\omega t)}+\frac{eV_1}{2ik_+}\frac{m}{\hbar^2}\left\{e^{ik_+x}\int_0^x dx'\,\phi(x')e^{i(k-k_+)x'}\right.$$

$$\left.-e^{-ik_+x}\int_0^x dx'\,\phi(x')e^{i(k+k_+)x'}+A_+e^{-ik_+x}\right\}e^{-i(\omega+\Omega)t}$$

$$+\frac{eV_1}{2ik_-}\frac{m}{\hbar^2}\left\{e^{ik_-x}\int_0^x dx'\,\phi(x')e^{i(k-k_-)x'}\right.$$

$$\left.-e^{-ik_-x}\int_0^x dx'\,\phi(x')e^{i(k+k_-)x'}+A_-e^{-ik_-x}\right\}e^{-i(\omega-\Omega)t},$$

where $\omega=\epsilon/\hbar$, $k=(2m\epsilon)^{1/2}/\hbar$, $k_\pm=[2m(\epsilon\pm\hbar\Omega)]^{1/2}/\hbar$, and

$$A_\pm=\frac{i}{(k+k_\pm)}\int_0^L dx\,\frac{d\phi}{dx}\,e^{i(k+k_\pm)x}.$$

Here $k_\pm$ are the wave vectors of the electrons in states with quasienergies $\epsilon\pm\hbar\Omega$; k_+ is always real, while k_- may be either real or imaginary. The latter case corresponds to localized states of the lower side band. The second solution, $\psi_<(x,t)$, is similar to the first, with k replaced by $-k$.

Using the wave functions $\psi_>$ and $\psi_<$ along with (1), we find the following expression for the current in the external circuit:

$$J(t)=\left[\frac{e^2}{h}A(\Omega)-i\,\frac{\Omega C}{2}\right]V_1e^{-i\Omega t}+\text{c.c.},$$

where $A(\Omega)$ is a dimensionless admittance associated with the ballistic conductance of the channel. This admittance can be thought of more conveniently as a function of the dimensionless frequency $\nu=\hbar\Omega/\epsilon_F$:

$$A(\nu)=\frac{1}{4\nu}\int_0^L dx\,\frac{d\phi}{dx}\int_0^L dy\,\frac{d\phi}{dy}\int_0^1\frac{dw}{\sqrt{w}}G(w,|x-y|), \tag{2}$$

where $w=\epsilon/\epsilon_F$,

$$G(w,\xi)=\frac{1}{k_F}\left\{\frac{(k_++k)^2}{k_+}\,e^{i(k_+-k)\xi}+\frac{(k_+-k)^2}{k_+}\,e^{i(k_++k)\xi}-\right.$$

$$\left.\frac{(k_-^*+k)^2}{k_-^*}\,e^{i(k-k_-^*)\xi}-\frac{(k_-^*-k)^2}{k_-^*}\,e^{-i(k_-^*+k)\xi}\right\},$$

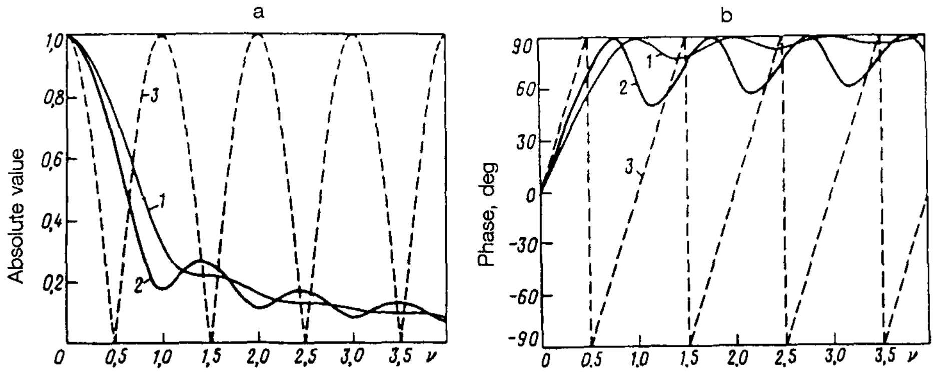

FIG. 1. Admittance versus the normalized frequency Ω/Ω_0 for three profiles of the alternating potential, $\phi(x)$, in the absence of a static barrier in the channel region. a—Absolute value of the admittance; b—phase of the admittance. 1) Linear potential; 2) knife-edge electrodes; 3) step potential.

$$k_F=(2m\epsilon_F)^{1/2}/\hbar.$$

The frequency dependence of the admittance is determined by the dimensionless parameter $b=Lk_F$ and by the potential profile $\phi(x)$. The case of most interest is the case $b\gg 1$. In the low-frequency region, with $b\nu^2\ll 8$, expression (2) simplifies:

$$A(\nu)\simeq\int_0^L dx\,\frac{d\phi}{dx}\int_0^L dy\,\frac{d\phi}{dy}\exp\left(i\,\frac{b\nu}{2}\,\frac{|x-y|}{L}\right). \tag{3}$$

It follows (first) that in the limit $\nu\to 0$ we have $A\to 1$. From (2) we thus obtain a transition to the known expression for the static conductance $2e^2/h$ for any profile of the time-varying potential. Second, we see a characteristic frequency Ω_0 in the frequency dependence of the admittance. This characteristic frequency is determined by the condition $b\nu/2=2\pi$; hence

$$\Omega_0=2\pi v_F/L,$$

where v_F is the electron Fermi velocity. The characteristic frequency is thus governed by the electron transit time in the channel.

The particular frequency dependence of the admittance is governed by the potential profile in the channel. Figure 1 shows the absolute value of the admittance and its phase as a function of the normalized frequency Ω/Ω_0 for three potential profiles $\phi(x)$, according to calculations from expression (3). Curve 1 was calculated for $\phi(x)=x/L$, corresponding to the geometry of a plane capacitor. Curve 2 corresponds to the case of knife-edge electrodes (the electrodes are two half-planes which lie in a common plane and which are separated by a gap L). In this case we find

$$\phi(x)=1-\frac{2}{\pi}\arccos\left(\frac{x}{L}\right).$$

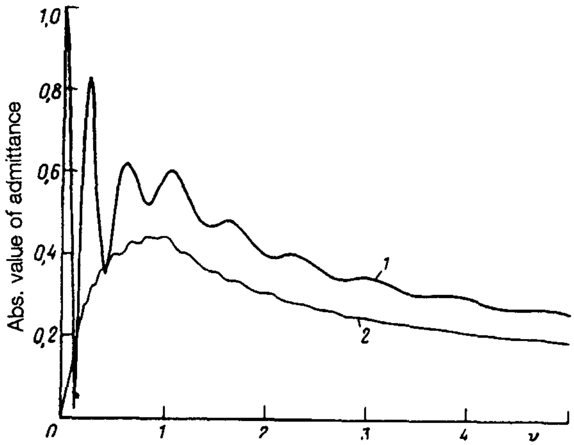

FIG. 2. Admittance versus the normalized frequency $\nu=\hbar\Omega/\epsilon_F$ for a step alternating potential with a barrier V_0 in the channel region. 1) $V_0=0.6\epsilon_F$; 2) $V_0=1.1\epsilon_F$. $b=30$.

Curve 3 corresponds to the potential distribution in the case of two steps at the ends of the channel:

$$\phi(x)=[\Theta(x)+\Theta(x-L)]/2.$$

It can be seen from Fig. 1 that the admittance oscillates as a function of the frequency with a characteristic period Ω_0.

The physical nature of the oscillations in the admittance corresponds to spatial resonances of charge density waves along the length of the channel. The interference of two electron waves $\exp[i(kx-\omega t)]$ from the fundamental band and a side band gives rise to a charge density wave $\exp\{i[(9k_+-k)x-\Omega t]\}$, where

$$k_+-k=\frac{2m}{\hbar}(\sqrt{\epsilon+\hbar\Omega}-\sqrt{\epsilon})\sim k_F\nu/2.$$

A resonance occurs under the condition $(k_+-k)L=2\pi n$, where n is an integer. This condition corresponds exactly to the condition $\Omega=n\Omega_0$, which, as we see from Fig. 1, is the frequency difference between the singularities of the admittance. The oscillations in the admittance increase in magnitude with increasing nonuniformity of the potential near the electrodes.

Figure 2 shows a frequency characteristic of the admittance over a broad frequency range including $\hbar\Omega>\epsilon_F$ for the case of a step profile of the alternating potential, with a static potential $V_0=\text{const}$ in the channel region. Curve 1 was calculated for $\epsilon_F>V_0$, and curve 2 for $\epsilon_F<V_0$. In the limit $\nu\rightarrow 0$ the admittance $A(\nu)$ tends toward $|t_0|^2$, where t_0 is the transmission coefficient of the static barrier for the electrons. This result agrees exactly with the Landauer formula.

In the case $\nu\ll 1$, we observe the oscillations described above; at $\nu>1$, they give way to a monotonic decay of $|A|\simeq\Omega_0/\Omega$. We thus have two characteristic frequencies in the problem: $\Omega=\Omega_0$ and $\Omega=\epsilon_F/\hbar$. The first is related to electron transit in the channel; it characterizes the oscillations of the admittance. The second characterizes the decay of the admittance at high frequencies due to the decrease in the probability for transitions of electrons into the side bands. Oscillations may occur under the condition $\Omega_0 \ll \epsilon_F/\hbar$, which is equivalent to the condition $Lk_F \gg 1$.

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[1)]Since the admittance is determined by the current response in the limit $V_1 \rightarrow 0$, the condition $\hbar\Omega \gg eV_1$ holds at any frequency.

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