Low-temperature features of the magnetocapacitance of 2D electron systems

V. B. Shikin
Institute of Solid State Physics, Russian Academy of Sciences, 141432 Chernogolovka, Moscow Region, Russia

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The differential magnetocapacitance of a 2D electron system in the form of a screened strip is calculated. A magnetic field is directed normal to the plane of the strip. The strength of this field corresponds to a nearly integer value of the filling factor.

In an interesting experimental study, Takaoka *et al.*[1] showed that, at its minima, the low-temperature magnetocapacitance of a two-dimensional (2D) electron system with a control electrode is proportional to the perimeter of the 2D system. This result is at odds with other calculations of the magnetocapacitance (see, for example, Refs. 2–4). Takaoka *et al.*[1] concluded, therefore, that magnetoelectric effects require a special analysis near minima of the magnetocapacitance. They believe that edge states may be contributing to the total capacitance of a bounded 2D system under conditions corresponding to integer values of the filling factor deep in the 2D system. The nature of these oscillations was not specified in Ref. 1; it was simply stated that the length scale of their localization near the boundary is far greater than the magnetic length.

In this letter we calculate the minimum magnetocapacitance of a 2D strip with a control electrode, utilizing a special electrostatics which holds in magnetized 2D systems with an integer filling factor.[5] The calculations confirm that the minimum capacitance is proportional to the perimeter of the 2D system. Arguments from Ref. 5 are systematically used in the literature to describe various magnetoelectric effects.[6,7] With the same degree of justification as in Refs. 5–7, we can naturally apply the electrostatics of Ref. 5 to the problem of the magnetocapacitance of 2D systems.

We consider a plane capacitor which has a large dimension along the Y axis, which has a width $2w$ along the X axis, and whose plates are separated by a distance $2d$. We place the origin of coordinates at the center of the capacitor. The magnetic field is directed along the Z axis. The system is immersed in a medium with a dielectric constant κ. The lower plate of the capacitor, which is in the $z=-d$ plane, is a 2D-electron system with an equilibrium electron density n_s, which corresponds, at the given magnetic field H, to the requirement that the filling factor ν have an integer value:

$$\nu=\pi l_H^2 n_s=1,2,3\ldots,\qquad l_H^2=c\hbar/eH. \tag{1}$$

Our task is to calculate the distribution of the additional electron density δn_1 along the 2D system when a potential difference V appears across the plates. The corresponding capacitance C is given by

$$C = dQ/dV, \quad Q = e\int_{-w}^{+w} \delta n_1(s)ds. \tag{2}$$

In the approximation of Ref. 5, the necessary equations are $(eV \ll \hbar\omega_c)$

$$\varphi_1(xz) + \varphi_2(xz)\Big|_{z=+d} = 0, \quad -w \leqslant x \leqslant +w, \tag{3}$$

$$\frac{\nu e}{h\omega_c}(\varphi_1'' + \varphi_2'')\Big|_{z=-d} = \delta n_1, \quad -w \leqslant x \leqslant +w, \tag{4}$$

$$\varphi_i' = d\varphi_i/dx, \quad \omega_c = eH/m_* c, \quad \varphi_1 + \varphi\Big|_{\substack{z=-d\\ x=\pm w}} = V,$$

$$\varphi_1'(xz) = \frac{2e}{\kappa}\int_{-w}^{+w} \frac{\delta n_1(s)(x-s)ds}{(x-s)^2 + (z+d)^2},$$

$$\varphi_2'(xz) = \frac{2e}{\kappa}\int_{-w}^{+w} \frac{\delta n_2(s)(x-s)ds}{(x-s)^2 + (z-d)^2}. \tag{5}$$

Here δn_1 and δn_2 are the distributions of excess charge densities along the 2D system and the control electrode, and φ_1 and φ_2 are corresponding potentials which arise because of the distributions δn_1 and δn_2. Requirement (3) is the standard condition that the control electrode be of an equipotential nature. Condition (4) from Ref. 5 determines the specific features of the behavior of the 2D system at integer values of ν. The quantity ω_c is the cyclotron frequency, and m_* is the effective mass of an electron.

In the limit $w \gg l \gg d$ we find the following expressions from (3)–(5):

$$\delta n_1 = l^2(\delta n_1'' + \delta n_2''), \quad l^2 = \nu l_H^2 d/a_b^*,$$

$$a_b^* = \kappa\hbar^2/m_* e^2, \quad \frac{2\pi e d}{\kappa}(\delta n_1 + \delta n_2)\Big|_{\pm w} = V, \tag{6}$$

$$\delta n_1 - \delta n_2 \simeq \frac{h\omega_c \kappa}{2e^2 \nu\sqrt{w^2 - x^2}}\int_{-w}^{+w} \frac{ds\sqrt{w^2 - s^2}}{x-s}\int_0^s \delta n_1 d\sigma,$$

$$\int_{-w}^{+w} (\delta n_1 - \delta n_2)ds = 0. \tag{7}$$

Here a_b^* is the effective first Bohr radius.

The requirement $l \gg d$, which we used in deriving Eqs. (6) and (7), is not necessary; it merely simplifies the calculations. For the parameter values from Ref. 1, it corresponds to the limit $\nu \gg 1$. In particular, in this case it is convenient to displace the origin of coordinates to one end of the strip (say, to the point $x = -w$) and to put Eqs. (6) and (7) in dimensionless form:

$$\frac{d^2}{d\xi^2}(\delta\tilde{n}_1 + \delta\tilde{n}_2) \simeq \delta n_1, \quad \xi = x/l,$$

$$\delta\tilde{n}_i=\delta n_i/n_s, \quad \delta\tilde{n}_1+\delta\tilde{n}_2\Big|_{\xi=0}=\tilde{V}, \quad \delta\tilde{n}_1+\delta\tilde{n}_2\Big|_{\infty}\to 0, \tag{8}$$

$$\tilde{V}=\kappa V/(2\pi e n_s d),$$

$$\delta\tilde{n}_1-\delta\tilde{n}_2\simeq\frac{\alpha}{\sqrt{\xi}}\int_0^\infty ds\,\frac{\sqrt{s}}{\xi-s}\int_0^s\delta\tilde{n}_1(\sigma)d\sigma,$$

$$\alpha=h\omega_c l/\nu e^2. \tag{9}$$

For the parameter values from Ref. 1 ($\kappa=12.3$, $m_*=0.07m_e$, $2d=1000$ Å and $n_s=2.8\times10^{11}$ cm^{-2}) and for a magnetic field $H\simeq10T$ (corresponding to $\nu\simeq1$), we have the estimates $l_H\simeq a_b^*\simeq10^{-6}$ cm, $l\simeq d$, and $\alpha\simeq1$. With increasing ν, the quantity l increases as ν^2, while the parameter α falls off as ν^{-1}. In the region $\nu\gg1$ we thus have $l\gg d$, as mentioned above; we also have $\alpha\ll1$.

Because the parameter α is a small value, we can seek a solution of (8) and (9) as a power series in α:

$$\delta\tilde{n}_1=\delta\tilde{n}_1^0+\alpha\delta\tilde{n}_1^{(1)}+..., \quad \delta\tilde{n}_2=\delta\tilde{n}_2^0+\alpha\delta\tilde{n}_2^{(1)}+.... \tag{10}$$

Substituting (10) into (8) and (9), we find

$$\delta\tilde{n}_1^0=\delta\tilde{n}_2^0, \quad 2\frac{d^2}{d\xi^2}\delta\tilde{n}_1^0=\delta\tilde{n}_1^0, \tag{11}$$

$$2\delta\tilde{n}_1^0\Big|_0=\tilde{V}, \quad \delta\tilde{n}_1^0\Big|_\infty\to0,$$

$$\delta\tilde{n}_1^{(1)}-\delta\tilde{n}_2^{(1)}=\frac{1}{\sqrt{\xi}}\int_0^\infty ds\,\frac{\sqrt{s}}{\xi-s}\int_0^s\delta\tilde{n}_1^0(\sigma)d\sigma, \tag{12}$$

$$\frac{d^2}{d\xi^2}(\delta\tilde{n}_1^{(1)}+\delta\tilde{n}_2^{(1)})=\delta\tilde{n}_1^{(1)}, \quad \delta\tilde{n}_1^{(1)}+\delta\tilde{n}_2^{(1)}\Big|_0=0,$$

$$\delta\tilde{n}_1^{(1)}+\delta\tilde{n}_2^{(1)}\Big|_{+\infty}\to0.$$

Restricting the discussion to the zeroth approximation, i.e., to Eqs. (11), we find

$$\delta\tilde{n}_1^0=\frac{\tilde{V}}{2}\exp\left(-\frac{\zeta}{\sqrt{2}}\right), \tag{13}$$

$$C_0=\frac{\sqrt{2}}{4\pi}\frac{\kappa l}{d}; \tag{14}$$

here C is from (2).

The results in (13) and (14) confirm the experimentally based assertion of Takaoka *et al.*[1] that excess charge localizes near the boundaries of a 2D system at an integer value of the filling factor ν. In the zeroth approximation in α, the length of this localization region is determined by the combination l from (6). The inequality $l \gg l_H$ corresponds to the data of Ref. 1 at a qualitative level, as does the behavior $l \propto \nu^2$ in the region $\nu > 1$.

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