Numerical simulation of two-pulse NMR in superfluid ^{3}He-*B*

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Two-pulse NMR has been simulated numerically in the spatially uniform approximation in the Leggett–Takagi two-fluid model. In the particular configuration considered, the first rf pulse, at the Larmor frequency Ω_L, sends the system into the regime of the long-lived Brinkman–Smith mode. The next rf pulse, at the sum or difference frequency $\Omega_L \pm \Omega_{\rm Ld}$, creates a nonequilibrium state. When a mechanism analogous to the mechanism of catastrophic relaxation operates, this nonequilibrium state may lead to the destruction of the coherent induction signal. © *1994 American Institute of Physics.*

According to the theory generally accepted at this point, liquid ^{3}He is a Fermi liquid at temperatures below 100 mK (Ref. 1). The physics of this Fermi liquid is characterized by molecular fields or Landau parameters. The latter can be found from data on the specific heat, compressibility, spin diffusion, and spin susceptibility. The presence of a superfluid transition based on p-wave pairing opens up some new possibilities for studying the Fermi structure of ^{3}He, because of the rich magnetic properties: In the Leggett–Takagi model, in addition to the total spin of the superfluid liquid, **S**, one distinguishes the spins of normal and superfluid components:

$$\mathbf{S}_p, \quad \mathbf{S}_q, \quad \mathbf{S}=\mathbf{S}_p+\mathbf{S}_q .$$

Bunkov *et al.*[2] utilized the properties of two-fluid spin dynamics to directly measure the Landau field corresponding to the parameter Z_0 at a temperature near $0.4T_c$ (i.e., well below the superfluid transition), at which the Landau field becomes approximately equal in magnitude to the external field,[3] and there is a resonance in the motion of the normal and superfluid spins. One might imagine that the spin dynamics in the experiment of Ref. 2 corresponded to two pulses: an ordinary rf pulse and a second rf pulse, which arose because of a crossing of the values of the external field and the Landau field. It is worthwhile to examine the situation in which the second pulse is created directly.

In research on spin dynamics in the two-fluid Leggett–Takagi model, a governing role is played by the quantity $\boldsymbol{\eta}=\mathbf{S}_p-\lambda\mathbf{S}$, where $\lambda=\chi_{p0}\cdot\chi_0^{-1}$ is the ratio of the superfluid and total susceptibilities, with Fermi-liquid corrections ignored. In an equilibrium state and also in the regime of the long-lived Brinkman–Smith mode of pulsed NMR, with a angular deflection of the total spin less than 104°, in the absence of an internal resonance of the precession of the normal and superfluid spins, the quantity $\boldsymbol{\eta}$ is small.[3] The idea of the experiment of Ref. 3 can be summarized as follows: Within the framework of two-fluid spin dynamics, one can distinguish the motions of the normal spin, the superfluid spin, and the total spin. Specifically, the total spin is rotating around the external field at the Larmor frequency $\Omega_L = \gamma H$, while the superfluid and normal spins are rotating around the Landau field $\mathbf{H}_{\rm Ld}$ at the frequency $\Omega_{\rm Ld} = \gamma H_{\rm Ld}$. The Landau field $\mathbf{H}_{\rm Ld}$ is itself rotating around the external field at the Larmor frequency Ω_L, along with the total spin. At temperatures at which the external field and the Landau field agree in magnitude, there is a resonance $\Omega_L = \Omega_{\rm Ld}$. This resonance should lead to a sharp increase in $\boldsymbol{\eta}$, an increase in the relaxation rate, and a collapse of the free-induction signal, which can be detected experimentally. In the numerical simulation of Ref. 5, in the spatially homogeneous approximation, this picture was confirmed. However, according to the results of Ref. 5 the internal resonance and the increase in $\boldsymbol{\eta}$ do not themselves lead to a dramatic increase in the relaxation rate. The Leggett–Takagi model should apparently incorporate some additional factor which influences the relaxation in the case in which there is an internal resonance. In the present letter, *we propose, in accordance with the results of Ref. 2,* that the existence of a resonance between the normal and superfluid spins, manifested in an increase in $\boldsymbol{\eta}$, leads to a sharp increase in the relaxation and to the disappearance of the free-induction signal. This resonance can thus be detected experimentally.

For a numerical simulation of the Leggett–Takagi spin-dynamics equations,[4] it is convenient to introduce the following dimensionless variables (here we are using the total susceptibility χ and the gyromagnetic ratio γ):

$$\mathbf{S}_R = \gamma^2 \chi^{-1} \Omega_L^{-1} \mathbf{S}, \quad \boldsymbol{\eta}_R = (1-\lambda)^{-1} \boldsymbol{\eta}, \quad t_R = \Omega_L t. \tag{1}$$

Below we discard the subscript R. The equations of motion for the total spin $\mathbf{S}$, for the vector $\boldsymbol{\eta}$, and for the order parameter, which is generally a complex 3×3 matrix and which we will represent as three column vectors, are

$$\frac{d}{dt}\mathbf{S} = \mathbf{S}\times\mathbf{H} + \mathbf{R}_D,$$

$$\frac{d}{dt}\boldsymbol{\eta} = \boldsymbol{\eta}\times\left[\mathbf{H} + \left(\frac{\hat{\chi}}{\hat{\chi}_0} - 1\right)\mathbf{S}\right] + \mathbf{R}_D - \frac{1}{\Omega_L \times \tau}\boldsymbol{\eta},$$

$$\frac{d}{dt}\mathbf{A}_j = \mathbf{A}_j \times \left[\mathbf{H} - \left(\mathbf{S} + \frac{1-\lambda}{\lambda}\frac{\hat{\chi}}{\hat{\chi}_0}\boldsymbol{\eta}\right)\right], \quad j = 1,2,3.$$

Here $\mathbf{R}_D$ is the dipole moment,[4] given by

$$\mathbf{R}_D = \frac{16}{15}\left(\frac{\Omega_{\rm Leg}}{\Omega_L}\right)^2 \sin\theta(\cos\theta + 1/4)\mathbf{n},$$

where $\Omega_{\rm Leg}$ is the Leggett frequency of the longitudinal NMR, θ nd $\mathbf{n}$ are the angle and axis of the order parameter for the B phase of ^{3}He, and $(\mathbf{A}_j)i = \mathbf{A}_{ij}$. The dissipation parameter here is $\Omega_L \cdot \tau$, where τ is the relaxation time for quasiparticles at the Fermi surface. The equations written above describe the spin dynamics in the spatially homogeneous approximation, i.e., disregarding the spin currents. In terms of the dimensionless variables in (1), the Larmor frequency is unity, while the Landau frequency is $\Omega_{\rm Ld} = \chi \cdot \chi_0^{-1} - 1$, where χ_0 is the susceptibility without Fermi-liquid corrections. We have

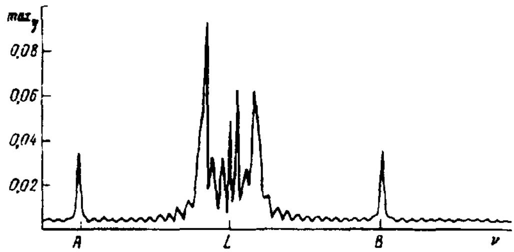

FIG. 1. $\Omega_{Leg}/\Omega_L=0.2$, $\chi/\chi_0=1.8$, $\lambda=0.99951$, $\Omega_L\cdot\tau=7000$. These results co rrespond to Ref. 5 and to temperatures and pressures near $T/T_c=0.2$ and 11 bar, respectively. The external field is characterized by $\Omega_L=1$ MHz. The amplitude of each of the rf pulses is 2% of the external field. The notation $A=|\Omega_L-\Omega_{Ld}|$, $L=\Omega_L$, $B=\Omega_L+\Omega_{Ld}$ is being used. The frequency (ν) of the second rf pulse is plotted along the x axis; the range is from 0 to 2.5 rad/s.

carried out a numerical study of these equations with the help of the Adams–Bashforth algorithm; some control calculations used some implicit algorithms: the Adams–Möller algorithm and the backward-differentiation algorithm. All the algorithms were of fourth-order accuracy.

We considered the standard pulsed-NMR configuration: The static external field **H** was applied along the z axis, and the rf pulses were applied in the perpendicular plane. The first pulse put the system in a state of precession, in the regime of the long-lived Brinkman–Smith model. This case corresponds experimentally to the HPD regime used in Ref. 2. After a delay time equal to one Larmor period, the second rf pulse was applied, and the system evolved under the influence of this pulse over 100 Larmor periods. The maximum value $\max_\eta$ of the vector $\boldsymbol{\eta}$ was fixed during the application of the second pulse. In this manner we obtained curves of $\max_\eta$ versus the frequency of the second pulse. These curves showed that there is a resonant increase in $\max_\eta$ at the sum and difference frequencies

$$\Omega_{Ld}+\Omega_L, \quad \Omega_{Ld}-\Omega_L$$

(Fig. 1). Comparison of the data found for various values of the dissipation constant $\Omega_L\cdot\tau$

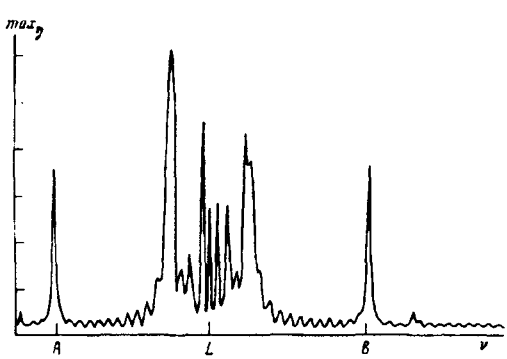

FIG. 2. The notation is the same as in Fig. 1. $\Omega_{Leg}/\Omega_L=0.3$, $\chi/\chi_0=1.8$, $\lambda=0.99951$, $\Omega_L\cdot\tau=4700$. The values for the governing dimensional quantities are the same as in Fig. 1, except that the external field is smaller by a factor of 1.5.

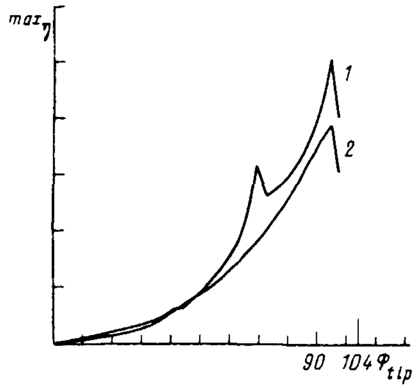

FIG. 3. $\Omega_{\text{Leg}}/\Omega_L=0.325$, $\chi/\chi_0=2.3$, $\lambda=0.9909303$. 1) $\Omega_L\cdot\tau=200$; 2) $\Omega_L\cdot\tau=120{,}9783$. Here ϕ_{tip} changes because of an increase in the amplitude of the first rf pulse from 0 to 3% of the external field.

shows that Leggett–Takagi dissipation can have a deleterious effect on the resonant nature of the dependence of $\max_\eta$ on the frequency of the second rf pulse. In order to study the effect, we should accordingly work at low temperatures: $0.4T_c$ and below. The strength of the static external field strongly influences the quality of the resonance picture, as can be seen by comparing Figs. 1 and 2. The reason is that in strong external fields the effect of the dipole torus is concealed. This entity is a governing factor in the dynamics of $\boldsymbol{\eta}$. On the other hand, small external fields lead to very strong nonlinear effects, which may also distort the picture of the resonance. Calculations show that the optimum value of the ratio $\Omega_{\text{Leg}}/\Omega_L$ is $\sim$0.2–0.3. Threshold effects associated with the values of the angular deviation of the magnetization, ϕ_{tip}, as a result of the application of the first rf pulse were not observed (Fig. 3).

In summary, by observing the effect of a second rf pulse on the quality of the free-induction signal one can apparently generate sum and difference frequencies corresponding to the Landau molecular field (to the parameter Z_0). A positive result of such an experiment would be one more piece of support for the hypothesis[2,3] that the sudden increase in the magnetization relaxation rate near $0.4T_c$ stems from the existence of an internal resonance between the superfluid and normal spins.

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[1] E. M. Lifshitz and L. P. Pitaevskii, *Statistical Physics* (Nauka, Moscow, 1978), Chapter I, Part II.
[2] Yu. M. Bunkov *et al.*, Phys. Rev. Lett. **68**, 600 (1992).
[3] Yu. M. Bunkov *et al.*, Physica B **165**, 675 (1990).
[4] A. J. Leggett *et al.*, Ann. Phys. **106**, 79 (1977).
[5] Yu. M. Bunkov and V. L. Golo, J. Low Temp. Phys. **90**, No. 3/4 (1993).

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